Mathematics Grade 10 WebBook - Teachers' Guide

An Introduction to Using the Mathematics Grade 10 WebBook

By:

Authors' and publisher's details omitted for anonymous review

ISBN: 978-1-920423-11-7

Table of Contents

Overview....6
Curriculum Overview....6
Curriculum Requirements and Objectives....7
According to the NCS:....8
Outcomes....10
Assessment....13
Tests....14
Projects / Investigations....15
Assignments....15
Examinations....16
Assessment Support....17
Baseline Assessment....17
Diagnostic Assessments....17
Other Short Diagnostic Tests....21
Exercises....22
Journal entries....23
Translations....24
Group Work....24
Mind Maps or Metacogs....25
Investigations....26
Orals....28
Chapter Contexts....29
Functions and Graphs....29
Number Patterns....29
Finance, Growth and Decay....29
Algebra....29
Products and Factors....29
Equations and Inequalities....30
Estimating Surds....30
Exponentials....30
Irrational Numbers & Rounding Off....30
Rational Numbers....30
Differential Calculus: Average Gradient....30
Probability....31
Euclidean Geometry and Measurement....31
Trigonometry....31
Analytical Geometry....31
Statistics....31
On the Web, Everyone can be a Scientist....32
Looking for Little Green Men....32
You are a Computer, too....33
FullMarks User Guide....35
What Can I do in FullMarks?....35

How do I do Each of These?....35
Access and Share Questions....35
Create Tests from Questions....36
Create Class Lists....36
Create Scoresheets....36
Analyse Learners' Performance....36
Rich Media....37
General Resources....37
First Layer: Educator Collaboration....37
Second Layer: Classroom Engagement....38
Third Layer: Beyond the Classroom....39
Embedded Content....39
Blog Posts....42
General Blogs....42
Maths Blogs....42
Chapter 1 - Review of Past Work....44
Solutions....44
Negative Numbers....44
Rearranging Equations....45
Real Numbers....45
Mathematical Symbols....45
End of Chapter Exercises....46
Chapter 2 - Functions and Graphs....47
Videos....47
Simulations....47
Solutions....48
Recap....48
Domain and Range....49
Intercepts....51
Parabolas....53
Graphs....54
Exponential Functions and Graphs....56
End of Chapter Exercises....57
Chapter 3 - Number Patterns....65
Videos....65
Solutions....66
Exercises....66
Chapter 4 - Finance....69
Videos....69
Solutions....70
Simple Interest....70
Compound Interest....71
Foreign Exchange....71
End Of Chapter Exercises....71
Chapter 5 - Rational Numbers....74

Videos....74
Solutions....74
Rational Numbers....74
Fractions....74
Repeated Decimal Notation....75
End of Chapter Exercises....75
Chapter 6 - Exponentials....76
Videos....76
Solutions....77
Applications using Law 1....77
Application using Law 2....77
Application using Law 3....78
Application using Law 4....78
Application using Law 5....78
Application using Law 6....78
End Of Chapter Exercises....78
Chapter 7 - Estimating Surds....80
Solutions....80
End of Chapter Exercises....80
Chapter 8 - Irrational Numbers and Rounding Off....81
Solutions....81
End of Chapter Exercises....81
Chapter 9 - Products and Factors....83
Videos....83
Solutions....84
Recap....84
Products....86
Factorising a Trinomial....88
Factorisation by Grouping....89
Simplification of Fractions....90
End of Chapter Exercises....91
Chapter 10 - Equations and Inequalities....93
Videos....93
Solutions....94
Solving Linear Equations....94
Solving Quadratic Equations....96
Solving Exponential Equations....98
Linear Inequalities....99
Simultaneous Equations....100
Solving Literal Equations....102
Mathematical Models....103
End of Chapter Exercises....104
Chapter 11 - Average Gradient....106
Solutions....106
End of Chapter Exercises....106

Chapter 12 - Probability..108
Videos..108
Solutions..108
Probability Models..108
Probability Identities..109
Mutually Exclusive Events..109
Random Experiments..111
End of Chapter Exercises..112
Chapter 13 - Geometry Basics..114
Videos..114
Solutions..115
Angles..115
Triangles..116
Exercises..117
Challenge Problem..119
Chapter 14 - Geometry..120
Videos 120
Solutions..121
Polygons – Mixed..121
Polygons..122
Surface Areas..124
Volume..124
Surface Areas and Volume:..125
Transformations..126
End of Chapter Exercises..127
Chapter 15 - Trigonometry..131
Presentations..131
Videos..131
Solutions..132
Finding Lengths..132
Applications of Trigonometric Functions..132
Graphs of Trigonometric Functions..133
End of Chapter Exercises..137
Chapter 16 - Analytical Geometry..144
Videos 144
Solutions..145
End of Chapter Exercises: 145
Chapter 17 - Statistics..151
Videos..151
Solutions..151
Exercises – Grouping Data..151
Exercises – Summarising Data..152
More Mean, Modal and Median Group Exercises..155
Exercises – Misuse of Statistics..156
Exercises 157

Overview

Curriculum Overview

Before 1994 there existed a number of education departments and subsequent curriculum according to the segregation that was so evident during the apartheid years. As a result, the curriculum itself became one of the political icons of freedom or suppression. Since then the government and political leaders have sought to try and develop one curriculum that is aligned with our national agenda of democratic freedom and equality for all, in fore-grounding the knowledge, skills and values our country believes our learners need to acquire and apply, in order to participate meaningfully in society as citizens of a free country. The National Curriculum Statement (NCS) of Grades R – 12 (DoE, 2011) therefore serves the purposes of:

- equipping learners, irrespective of their socio-economic background, race, gender, physical ability or intellectual ability, with the knowledge, skills and values necessary for self-fulfilment, and meaningful participation in society as citizens of a free country;
- providing access to higher education;
- facilitating the transition of learners from education institutions to the workplace; and
- providing employers with a sufficient profile of a learner's competencies.

Although elevated to the status of political icon, the curriculum remains a tool that requires the skill of an educator in interpreting and operationalising this tool within the classroom. The curriculum itself cannot accomplish the purposes outlined above without the community of curriculum specialists, material developers, educators and assessors contributing to and supporting the process, of the intended curriculum becoming the implemented curriculum. A curriculum can succeed or fail, depending on its implementation, despite its intended principles or potential on paper. It is therefore important that stakeholders of the curriculum are familiar with and aligned to the following principles that the NCS is based on:

Principle	Implementation
Social Transformation	Redressing imbalances of the past. Providing equal opportunities for all.
Active and Critical Learning	Encouraging an active and critical approach to learning. Avoiding excessive rote and uncritical learning of given truths.
High Knowledge and Skills	Learners achieve minimum standards of knowledge and skills specified for each grade in each subject.
Progression	Content and context shows progression from simple to complex.

Social and Environmental Justice and Human Rights	These practices as defined in the Constitution are infused into the teaching and learning of each of the subjects.
Valuing Indigenous Knowledge Systems	Acknowledging the rich history and heritage of this country.
Credibility, Quality and Efficiency	Providing an education that is globally comparable in quality.

This guide is intended to add value and insight to the existing National Curriculum for Grade 10 Mathematics, in line with its purposes and principles. It is hoped that this will assist you as the educator in optimising the implementation of the intended curriculum.

Curriculum Requirements and Objectives

The main objectives of the curriculum relate to the learners that emerge from our educational system. While educators are the most important stakeholders in the implementation of the intended curriculum, the quality of learner coming through this curriculum will be evidence of the actual attained curriculum from what was intended and then implemented.

These purposes and principles aim to produce learners that are able to:

- identify and solve problems and make decisions using critical and creative thinking;
- work effectively as individuals and with others as members of a team;
- organise and manage themselves and their activities responsibly and effectively;
- collect, analyse, organise and critically evaluate information;
- communicate effectively using visual, symbolic and/or language skills in various modes;
- use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
- demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

The above points can be summarised as an independent learner who can think critically and analytically, while also being able to work effectively with members of a team and identify and solve problems through effective decision making. This is also the outcome of what educational research terms the “reformed” approach rather than the “traditional” approach many educators are more accustomed to. Traditional practices have their role and cannot be totally abandoned in favour of only reform practices. However, in order to produce more independent and mathematical thinkers, the reform ideology needs to be more embraced by educators within their instructional behaviour. Here is a table that can guide you to identify your dominant instructional practice and try to assist you in adjusting it (if necessary) to be more balanced and in line with the reform approach being suggested by the NCS.

	Traditional Versus Reform Practices
Values	**Traditional** – values content, correctness of learners' responses and mathematical validity of methods. **Reform** – values finding patterns, making connections, communicating mathematically and problem-solving.
Teaching Methods	**Traditional** – expository, transmission, lots of drill and practice, step by step mastery of algorithms. **Reform** – hands-on guided discovery methods, exploration, modelling. High level reasoning processes are central.
Grouping Learners	**Traditional** – dominantly same grouping approaches. **Reform** – dominantly mixed grouping and abilities.

The subject of mathematics, by the nature of the discipline, provides ample opportunities to meet the reformed objectives. In doing so, the definition of mathematics needs to be understood and embraced by educators involved in the teaching and the learning of the subject. In research it has been well documented that, as educators, our conceptions of what mathematics is, has an influence on our approach to the teaching and learning of the subject.

Three possible views of mathematics can be presented. The instrumentalist view of mathematics assumes the stance that mathematics is an accumulation of facts, rules and skills that need to be used as a means to an end, without there necessarily being any relation between these components. The Platonist view of mathematics sees the subject as a static but unified body of certain knowledge, in which mathematics is discovered rather than created. The problem solving view of mathematics is a dynamic, continually expanding and evolving field of human creation and invention that is in itself a cultural product. Thus mathematics is viewed as a process of enquiry, not a finished product. The results remain constantly open to revision. It is suggested that a hierarchical order exists within these three views, placing the instrumentalist view at the lowest level and the problem solving view at the highest.

According to the NCS:

Mathematics is the study of quantity, structure, space and change. Mathematicians seek out patterns, formulate new conjectures, and establish axiomatic systems by rigorous deduction from appropriately chosen axioms and definitions. Mathematics is a distinctly human activity practised by all cultures, for thousands of years. Mathematical problem solving enables us to understand the world (physical, social and economic) around us, and, most of all, to teach us to think creatively.

This corresponds well to the problem solving view of mathematics and may challenge some of our instrumentalist or Platonistic views of mathematics as a static body of knowledge of accumulated facts, rules and skills to be learnt and applied. The NCS is trying to discourage such an approach and encourage mathematics educators to dynamically and creatively involve their learners as mathematicians engaged in a process of study, understanding, reasoning, problem solving and communicating mathematically.

Below is a check list that can guide you in actively designing your lessons in an attempt to embrace the definition of mathematics from the NCS and move towards a problem solving conception of the subject. Adopting such an approach to the teaching and learning of mathematics will in turn contribute to the intended curriculum being properly implemented and attained through the quality of learners coming out of the education system.

Practice	Example
Learners engage in solving contextual problems related to their lives that require them to interpret a problem and then find a suitable mathematical solution.	Learners are asked to work out which bus service is the cheapest given the fares they charge and the distance they want to travel.
Learners engage in solving problems of a purely mathematical nature, which require higher order thinking and application of knowledge (non-routine problems).	Learners are required to draw a graph; they have not yet been given a specific technique on how to draw (for example a parabola), but have learnt to use the table method to draw straight-line graphs.
Learners are given opportunities to negotiate meaning.	Learners discuss their understanding of concepts and strategies for solving problems with each other and the educator.
Learners are shown and required to represent situations in various but equivalent ways (mathematical modelling).	Learners represent data using a graph, a table and a formula to represent the same data.
Learners individually do mathematical investigations in class, guided by the educator where necessary.	Each learner is given a paper containing the mathematical problem (for instance to find the number of prime numbers less than 50) that needs to be investigated and the solution needs to be written up. Learners work independently.
Learners work together as a group/team to investigate or solve a mathematical problem.	A group is given the task of working together to solve a problem that requires them investigating patterns and working through data to make conjectures and find a formula for the pattern.
Learners do drill and practice exercises to consolidate the learning of concepts and to master various skills.	Completing an exercise requiring routine procedures.
Learners are given opportunities to see the interrelatedness of the mathematics and to see how the different outcomes are related and connected.	While learners work through geometry problems, they are encouraged to make use of algebra.
Learners are required to pose problems for their educator and peer learners.	Learners are asked to make up an algebraic word problem (for which they also know the solution) for the person sitting next to them to solve.

Outcomes

Summary of topics, outcomes and their relevance:

1. Functions – linear, quadratic, exponential, rational		
Outcome		**Relevance**
10.1.1	Relationships between variables in terms of graphical, verbal and symbolic representations of functions (tables, graphs, words and formulae).	Functions form a core part of learners' mathematical understanding and reasoning processes in algebra. This is also an excellent opportunity for contextual mathematical modelling questions.
10.1.2	Generating graphs and generalising effects of parameters of vertical shifts and stretches and reflections about the x-axis.	
10.1.3	Problem solving and graph work involving prescribed functions.	

2. Number Patterns, Sequences and Series		
Outcome		**Relevance**
10.2.1	Number patterns with constant difference.	Much of mathematics revolves around the identification of patterns.

3. Finance, Growth and Decay		
Outcome		**Relevance**
10.3.1	Use simple and compound growth formulae.	The mathematics of finance is very relevant to daily and long-term financial decisions learners will need to make in terms of investing, taking loans, saving and understanding exchange rates and their influence more globally.
10.3.2	Implications of fluctuating exchange rates.	

4. Algebra		
Outcome		**Relevance**
10.4.1	Identifying and converting forms of rational numbers. Working with simple surds that are not rational.	Algebra provides the basis for mathematics learners to move from numerical calculations to generalising operations, simplifying expressions, solving equations and using graphs and inequalities in solving contextual
10.4.2	Working with laws of integral exponents. Establish between which two integers a simple surd lies.	

	Appropriately rounding off real numbers.	problems.
10.4.3	Manipulating and simplifying algebraic expressions (including multiplication and factorisation).	
10.4.4	Solving linear, quadratic and exponential equations. Solving linear inequalities in one and two variables algebraically and graphically.	

5. Differential Calculus		
Outcome		**Relevance**
10.5.1	Investigate average rate of change between two independent values of a function.	The central aspect of rate of change to differential calculus is a basis to further understanding of limits, gradients and calculations and formulae necessary for work in engineering fields, e.g. designing roads, bridges etc.

6. Probability		
Outcome		**Relevance**
10.6.1	Compare relative frequency and theoretical probability. Use Venn diagrams to solve probability problems. Mutually exclusive and complementary events. Identity for any two events A and B.	This topic is helpful in developing good logical reasoning in learners and for educating them in terms of real-life issues such as gambling and the possible pitfalls thereof.

7. Euclidean Geometry and Measurement		
Outcome		**Relevance**
10.7.1	Investigate, form and try to prove conjectures about properties of special triangles, quadrilaterals and other polygons. Disprove false conjectures using counter-examples. Investigate alternative definitions of various polygons.	The thinking processes and mathematical skills of proving conjectures and identifying false conjectures is more the relevance here than the actual content studied. The surface area and volume content studied in real-life contexts of designing kitchens, tiling and painting rooms, designing packages, etc.
10.7.2	Solve problems involving surface area and volumes of solids and combinations thereof.	

		is relevant to the current and future lives of learners.

8. Trigonometry		
Outcome		**Relevance**
10.8.1	Definitions of trig functions. Derive values for special angles. Take note of names for reciprocal functions.	Trigonometry has several uses within society, including within navigation, music, geographical locations and building design and construction.
10.8.2	Solve problems in 2 dimensions.	
10.8.3	Extend definition of basic trig functions to all four quadrants and know graphs of these functions.	
10.8.4	Investigate and know the effects of a and q on the graphs of basic trig functions.	

9. Analytical Geometry		
Outcome		**Relevance**
10.9.1	Represent geometric figures on a Cartesian coordinate system. For any two points, derive and apply formula for calculating distance, gradient of line segment and coordinates of mid-point.	This section provides a further application point for learners' algebraic and trigonometric interaction with the Cartesian plane. Artists and design and layout industries often draw on the content and thought processes of this mathematical topic.

10. Statistics		
Outcome		**Relevance**
10.10.1	Collect, organise and interpret univariate numerical data to determine mean, median, mode, percentiles, quartiles, deciles, interquartile and semi-interquartile range.	Citizens are daily confronted with interpreting data presented from the media. Often this data may be biased or misrepresented within a certain context. In any type of research, data collection and handling is a core feature. This topic also educates learners to become more socially and politically educated with regards to the media.
10.10.2	Identify possible sources of bias and errors in measurements.	

Mathematics educators also need to ensure that the following important specific aims and general principles are applied in mathematics activities across all grades:

- Calculators should only be used to perform standard numerical computations and verify calculations done by hand.
- Real-life problems should be incorporated into all sections to keep mathematical modelling as an important focal point of the curriculum.
- Investigations give learners the opportunity to develop their ability to be more methodical, to generalise and to make and justify and/or prove conjectures.
- Appropriate approximation and rounding skills should be taught and continuously included and encouraged in activities.
- The history of mathematics should be incorporated into projects and tasks where possible, to illustrate the human aspect and developing nature of mathematics.
- Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues where possible.
- Conceptual understanding of when and why should also feature in problem types.
- Mixed ability teaching requires educators to challenge able learners and provide remedial support where necessary.
- Misconceptions exposed by assessment need to be dealt with and rectified by questions designed by educators.
- Problem solving and cognitive development should be central to all mathematics teaching and learning so that learners can apply the knowledge effectively.

Assessment

"Educator assessment is part of everyday teaching and learning in the classroom. Educators discuss with learners, guide their work, ask and answer questions, observe, help, encourage and challenge. In addition, they mark and review written and other kinds of work. Through these activities they are continually finding out about their learners' capabilities and achievements. This knowledge then informs plans for future work. It is this continuous process that makes up educator assessment. It should not be seen as a separate activity necessarily requiring the use of extra tasks or tests."

As the quote above suggests, assessment should be incorporated as part of the classroom practice, rather than as a separate activity. Research during the past ten years indicates that learners get a sense of what they do and do not know, what they might do about this and how they feel about it, from frequent and regular classroom assessment and educator feedback. The educator's perceptions of and approach to assessment (both formal and informal assessment) can have an influence on the classroom culture that is created with regard to the learners' expectations of and performance in assessment tasks. Literature on classroom assessment distinguishes between two different purposes of assessment; assessment **of** learning and assessment **for** learning.

Assessment **of** learning tends to be a more formal assessment and assesses how much learners have learnt or understood at a particular point in the annual teaching plan. The NCS provides comprehensive guidelines on the types of and amount of formal assessment that needs to take place within the teaching year to make up the school-based assessment mark. The school-based assessment mark contributes 25% of the final percentage of a learner's promotion mark, while the end-of-year examination constitutes the other 75% of the annual promotion mark. Learners are expected to have 7 formal assessment tasks for their school-based assessment mark. The number of tasks and their weighting in the Grade 10 Mathematics curriculum is summarised below:

		Tasks	Weight (%)
School-Based Assessment	Term 1	Test Project/Investigation	10 20
	Term 2	Assignment/Test Examination	10 30
	Term 3	Test Test	10 10
	Term 4	Test	10
School-Based Assessment Mark			100
School-Based Assessment Mark (as a % of Promotion Mark)			25 %
End-of-Year Examination			75 %
Promotion Mark			100 %

The following provides a brief explanation of each of the assessment tasks included in the assessment programme above.

Tests

All mathematics educators are familiar with this form of formal assessment. Tests include a variety of items/questions covering the topics that have been taught prior to the test. The new NCS also stipulates that mathematics tests should include questions that cover the following four types of cognitive levels in the stipulated weightings:

Cognitive Levels	Description	Weighting (%)
Knowledge	Estimation and appropriate rounding of numbers. Proofs of prescribed theorems. Derivation of formulae. Straight recall. Identification and direct use of formula on information sheet (no changing of the subject). Use of mathematical facts. Appropriate use of mathematical vocabulary.	20
Routine Procedures	Perform well known procedures. Simple applications and calculations. Derivation from given information. Identification and use (including changing the subject) of correct formula. Questions generally similar to those done in class.	45
Complex Procedures	Problems involve complex calculations and/or	25

	higher reasoning. There is often not an obvious route to the solution. Problems need not be based on real world context. Could involve making significant connections between different representations. Require conceptual understanding.	
Problem Solving	Unseen, non-routine problems (which are not necessarily difficult). Higher order understanding and processes are often involved. Might require the ability to break the problem down into its constituent parts.	10

The breakdown of the tests over the four terms is summarised from the NCS assessment programme as follows:

Term 1: One test of at least 50 marks, and one hour or two/three tests of at least 40 minutes each.

Term 2: Either one test (of at least 50 marks) or an assignment.

Term 3: Two tests, each of at least 50 marks and one hour.

Term 4: One test of at least 50 marks .

Projects / Investigations

Investigations and projects consist of open-ended questions that initiate and expand thought processes. Acquiring and developing problem-solving skills are an essential part of doing investigations and projects. These tasks provide learners with the opportunity to investigate, gather information, tabulate results, make conjectures and justify or prove these conjectures. Examples of investigations and projects and possible marking rubrics are provided in the next section on assessment support. The NCS assessment programme indicates that only one project or investigation (of at least 50 marks) should be included per year. Although the project/investigation is scheduled in the assessment programme for the first term, it could also be done in the second term.

Assignments

The NCS includes the following tasks as good examples of assignments:

- Open book test
- Translation task
- Error spotting and correction
- Shorter investigation
- Journal entry
- Mind-map (also known as a metacog)
- Olympiad (first round)
- Mathematics tutorial on an entire topic
- Mathematics tutorial on more complex/problem solving questions

The NCS assessment programme requires one assignment in term 2 (of at least 50 marks) which could also be a combination of some of the suggested examples above. More information on these suggested examples of assignments and possible rubrics are provided in the following section on assessment support.

Examinations

Educators are also all familiar with this summative form of assessment that is usually completed twice a year: mid-year examinations and end-of-year examinations. These are similar to the tests but cover a wider range of topics completed prior to each examination. The NCS stipulates that each examination should also cover the four cognitive levels according to their recommended weightings as summarised in the section above on tests. The following table summarises the requirements and information from the NCS for the two examinations.

Examination	Marks	Breakdown	Content and Mark Distribution
Mid-Year Exam	100 50 + 50	One paper: 2 hours **or** Two papers: each of 1 hour	Topics completed
End-of-Year Exam	100 +	Paper 1: 2 hours	Number patterns (±10) Algebraic expressions, equations and inequalities (±25) Functions (±35) Exponents (±10) Finance (±10) Probability (±10)
	100	Paper 2: 2 hours	Trigonometry (±45) Analytical geometry (±15) Euclidean geometry and measurement (±25) Statistics (±15)

In the annual teaching plan summary of the NCS in Mathematics for Grade 10, the pace setter section provides a detailed model of the suggested topics to be covered each week of each term and the accompanying formal assessment.

Assessment **for** learning tends to be more informal and focuses on using assessment in and of daily classroom activities that can include:

- Marking homework
- Baseline assessments
- Diagnostic assessments
- Group work
- Class discussions
- Oral presentations

- Self-assessment
- Peer-assessment

These activities are expanded on in the next section on assessment support and suggested marking rubrics are provided. Where formal assessment tends to restrict the learner to written assessment tasks, the informal assessment is necessary to evaluate and encourage the progress of the learners in their verbal mathematical reasoning and communication skills. It also provides a less formal assessment environment that allows learners to openly and honestly assess themselves and each other, taking responsibility for their own learning, without the heavy weighting of the performance (or mark) component. The assessment **for** learning tasks should be included in the classroom activities at least once a week (as part of a lesson) to ensure that the educator is able to continuously evaluate the learners' understanding of the topics covered as well as the effectiveness, and identify any possible deficiencies in his or her own teaching of the topics.

Assessment Support

A selection of explanations, examples and suggested marking rubrics for the assessment **of** learning (formal) and the assessment **for** learning (informal) forms of assessment discussed in the preceding section are provided in this section.

Baseline Assessment

Baseline assessment is a means of establishing:

- What prior knowledge a learner possesses
- What the extent of knowledge is that they have regarding a specific learning area
- The level they demonstrate regarding various skills and applications
- The learner's level of understanding of various learning areas

It is helpful to educators in order to assist them in taking learners from their individual point of departure to a more advanced level and to thus make progress. This also helps avoid large "gaps" developing in the learners' knowledge as the learner moves through the education system. Outcomes-based education is a more learner-centered approach than we are used to in South Africa, and therefore the emphasis should now be on the level of each individual learner rather than that of the whole class.

The baseline assessments also act as a gauge to enable learners to take more responsibility for their own learning and to view their own progress. In the traditional assessment system, the weaker learners often drop from a 40% average in the first term to a 30% average in the fourth term due to an increase in workload, thus demonstrating no obvious progress. Baseline assessment, however, allows for an initial assigning of levels which can be improved upon as the learner progresses through a section of work and shows greater knowledge, understanding and skill in that area.

Diagnostic Assessments

These are used to specifically find out if any learning difficulties or problems exist within a section of work in order to provide the learner with appropriate additional help and guidance. The assessment helps the educator and the learner identify problem areas, misunderstandings, misconceptions and incorrect use and interpretation of notation.

Some points to keep in mind:

- Try not to test too many concepts within one diagnostic assessment.
- Be selective in the type of questions you choose.
- Diagnostic assessments need to be designed with a certain structure in mind. As an educator, you should decide exactly what outcomes you will be assessing and structure the content of the assessment accordingly.
- The assessment is marked differently to other tests in that the mark is not the focus but rather the type of mistakes the learner has made.

An example of an understanding rubric for educators to record results is provided below:

0	-	indicates that the learner has not grasped the concept at all and that there appears to be a fundamental mathematical problem.
1	-	indicates that the learner has gained some idea of the content, but is not demonstrating an understanding of the notation and concept.
2	-	indicates evidence of some understanding by the learner but further consolidation is still required.
3	-	indicates clear evidence that the learner has understood the concept and is using the notation correctly.

An example of a diagnostic assessment to evaluate learners' proficiency in calculator skills is provided below. There is a component of self-assessment as well as a component on educator assessment and how to group the various questions to diagnose any gaps or problems with learners' calculator skills.

Calculator worksheet - diagnostic skills assessment

Question 1

Calculate:

a) 242 + 63 ______________

b) 2 – 36 x (114 + 25) ______________

c) $\sqrt{144+25}$ ______________

d) $\sqrt[4]{729}$ ______________

e) –312 + 6 + 879 – 321 + 18 901 ______________

Question 2

Calculate:

a) $\frac{2}{7}+\frac{1}{3}$ = __________

b) $2\frac{1}{5}-\frac{2}{9}$ = __________

c) $-2\frac{5}{6}+\frac{3}{8}$ = __________

d) $4-\frac{3}{4}\times\frac{5}{7}$ = __________

e) $\left(\frac{9}{10}-\frac{8}{9}\right)\div\frac{3}{5}$ = __________

f) $2\times\left(\frac{4}{5}\right)^2-\left(\frac{19}{25}\right)$ = __________

g) $\sqrt{\frac{9}{4}-\frac{4}{16}}$ = __________

Self-Assessment Rubric: Name: ____________________

Question	Answer	√	X	If X, write down sequence of keys pressed
1a)				
1b)				
1c)				
1d)				
1e)				
Subtotal				
2a)				
2b)				
2c)				
2d)				
2e)				
2f)				
2g)				
Subtotal				
Total				

Educator Assessment Rubric:

Type of Skill	Competent	Needs Practice	Problem
Raising to a Power			
Finding a Root			
Calculations with Fractions			
Brackets and Order of Operations			
Estimation and Mental Control			

Guidelines for Calculator Skills Assessment:

Type of Skill	Sub-Division	Questions
Raising to a Power	Squaring and cubing Higher order powers	1a, 2f 1b
Finding a Root	Square and cube roots Higher order roots	1c, 2g 1d
Calculations with Fractions	Basic operations Mixed numbers Negative numbers Squaring fractions Square rooting fractions	2a, 2d 2b, 2c 1e, 2c 2f 2g
Brackets and Order of Operations	Correct use of brackets or order of operations	1b, 1c, 2e, 2f, 2g
Estimation and Mental Control	Overall	All

Suggested guideline to allocation of overall levels

Level 1

- Learner is able to do basic operations on calculator.
- Learner is able to do simple calculations involving fractions.
- Learner does not display sufficient mental estimation and control techniques.

Level 2

- Learner is able to do basic operations on calculator.
- Learner is able to square and cube whole numbers as well as find square and cube roots of numbers.
- Learner is able to do simple calculations involving fractions as well as correctly execute

calculations involving mixed numbers.
- Learner displays some degree of mental estimation awareness.

Level 3
- Learner is able to do basic operations on calculator.
- Learner is able to square and cube rational numbers as well as find square and cube roots of numbers.
- Learner is also able to calculate higher order powers and roots.
- Learner is able to do simple calculations involving fractions as well as correctly execute calculations involving mixed numbers.
- Learner works correctly with negative numbers.
- Learner is able to use brackets in certain calculations but has still not fully understood the order of operations that the calculator has been programmed to execute, hence the need for brackets.
- Learner is able to identify possible errors and problems in their calculations but needs assistance solving the problem.

Level 4
- Learner is able to do basic operations on calculator.
- Learner is able to square and cube rational numbers as well as find square and cube roots.
- Learner is also able to calculate higher order powers and roots.
- Learner is able to do simple calculations involving fractions as well as correctly execute calculations involving mixed numbers.
- Learner works correctly with negative numbers.
- Learner is able to work with brackets correctly and understands the need and use of brackets and the "= key" in certain calculations due to the nature of a scientific calculator.
- Learner is able to identify possible errors and problems in their calculations and to find solutions to these in order to arrive at a "more viable" answer.

Other Short Diagnostic Tests

These are short tests that assess small quantities of recall knowledge and application ability on a day-to-day basis. Such tests could include questions on one or a combination of the following:
- Definitions
- Theorems
- Riders (geometry)
- Formulae
- Applications
- Combination questions

Here is a selection of model questions that can be used at Grade 10 level to make up short diagnostic tests. They can be marked according to a memorandum drawn up by the educator.

Geometry

1.Points A (-5 ; -3), B (-1 ; 2) and C (9 ; -6) are the vertices of ΔABC.

a) Calculate the gradients of AB and BC and hence show that angle ABC is equal to 90° . (5)

b) State the distance formula and use it to calculate the lengths of the sides AB, BC and AC of ΔABC. (Leave your answers in surd form). (5)

Algebra

1. Write down the formal definition of an exponent as well as the exponent laws for integral exponents. (6)

2. Simplify: $\frac{2x^4 y^8 z^3}{4xy} \times \frac{x^7}{y^3 z^0}$ (4)

Trigonometry

1. A jet leaves an airport and travels 578 km in a direction of 50° E of N. The pilot then changes direction and travels 321 km 10° W of N.

a) How far away from the airport is the jet? (To the nearest kilometre) (5)

b) Determine the jet's bearing from the airport. (5)

Exercises

This entails any work from the textbook or other source that is given to the learner, by the educator, to complete either in class or at home. Educators should encourage learners not to copy each other's work and be vigilant when controlling this work. It is suggested that such work be marked/controlled by a check list (below) to speed up the process for the educator.

The marks obtained by the learner for a specific piece of work need not be based on correct and/or incorrect answers but preferably on the following:

- the effort of the learner to produce answers.
- the quality of the corrections of work that was previously incorrect.
- the ability of the learner to explain the content of some selected examples (whether in writing or orally).

The following rubric can be used to assess exercises done in class or as homework:

Criteria	**Performance Indicators**		
Work Done	2 All the work	1 Partially completed	0 No work done
Work Neatly Done	2 Work neatly done	1 Some work not neatly done	0 Messy and muddled
Corrections Done	2 All corrections done consistently	1 At least half of the corrections done	0 No corrections done
Correct Mathematical Method	2 Consistently	1 Sometimes	0 Never
Understanding of Mathematical Techniques and Processes	2 Can explain concepts and processes precisely	1 Explanations are ambiguous or not focused	0 Explanations are confusing or irrelevant

Journal entries

A journal entry is an attempt by a learner to express in the written word what is happening in Mathematics. It is important to be able to articulate a mathematical problem, and its solution in the written word.

This can be done in a number of different ways:

- Today in Maths we learnt _______________________
- Write a letter to a friend, who has been sick, explaining what was done in class today.
- Explain the thought process behind trying to solve a particular maths problem, e.g. sketch the graph of $y=x^2-2x^2+1$ and explain how to sketch such a graph.
- Give a solution to a problem, decide whether it is correct and if not, explain the possible difficulties experienced by the person who wrote the incorrect solution.

A journal is an invaluable tool that enables the educator to identify any mathematical misconceptions of the learners. The marking of this kind of exercise can be seen as subjective but a marking rubric can simplify the task.

The following rubric can be used to mark journal entries. The learners must be given the marking rubric before the task is done.

Task	**Competent (2 Marks)**	**Still Developing (1 Mark)**	**Not Yet Developed (0 Marks)**
Completion in Time Limit?			
Correctness of the Explanation?			
Correct and Relevant use of Mathematical Language?			
Is the Mathematics Correct?			
Has the Concept Been Interpreted Correctly?			

Translations

Translations assess the learner's ability to translate from words into mathematical notation or to give an explanation of mathematical concepts in words. Often when learners can use mathematical language and notation correctly, they demonstrate a greater understanding of the concepts.

For example:
Write the letter of the correct expression next to the matching number:

x increased by 10	a)	xy
The product of x and y	b)	x^2
The sum of a certain number and	c)	x^2
double that number	d)	29x
Half of a certain number multiplied by itself	e)	½ x 2
Two less than x	f)	x + x + 2
A certain number multiplied by itself	g)	x 2
Two consecutive even numbers	h)	x 29
x + x + x + …… to 29 terms	i)	x + 2x
x.x.x.x.x….. to 29 factors	j)	x + 10
A certain number divided by 2		

Group Work

One of the principles in the NCS is to produce learners who are able to work effectively within a group. Learners generally find this difficult to do. Learners need to be encouraged to work within small groups. Very often it is while learning under peer assistance that a better understanding of concepts and processes is reached. Clever learners usually battle with this sort of task, and yet it is important that they learn how to assist and communicate effectively with other learners.

Mind Maps or Metacogs

A metacog or “mind map” is a useful tool. It helps to associate ideas and make connections that would otherwise be too unrelated to be linked. A metacog can be used at the beginning or end of a section of work in order to give learners an overall perspective of the work covered, or as a way of recalling a section already completed. It must be emphasised that it is not a summary. Whichever way you use it, it is a way in which a learner is given the opportunity of doing research in a particular field and can show that he/she has an understanding of the required section.

This is an open book form of assessment and learners may use any material they feel will assist them. It is suggested that this activity be practised, using other topics, before a test metacog is submitted for portfolio assessment purposes.

On completion of the metacog, learners must be able to answer insightful questions on the metacog. This is what sets it apart from being just a summary of a section of work. Learners must refer to their metacog when answering the questions, but may not refer to any reference material. Below are some guidelines to give to learners to adhere to when constructing a metacog as well as two examples to help you get learners started. A marking rubric is also provided. This should be made available to learners before they start constructing their metacogs. On the next page is a model question for a metacog, accompanied by some sample questions that can be asked within the context of doing a metacog about analytical geometry.

A basic metacog is drawn in the following way:

- Write the title/topic of the subject in the centre of the page and draw a circle around it.
- For the first main heading of the subject, draw a line out from the circle in any direction, and write the heading above or below the line.
- For sub-headings of the main heading, draw lines out from the first line for each subheading and label each one.
- For individual facts, draw lines out from the appropriate heading line.

Metacogs are one’s own property. Once a person understands how to assemble the basic structure they can develop their own coding and conventions to take things further, for example to show linkages between facts. The following suggestions may assist educators and learners to enhance the effectiveness of their metacogs:

- Use single words or simple phrases for information. Excess words just clutter the metacog and take extra time to write down.
- Print words – joined up or indistinct writing can be more difficult to read and less attractive to look at.
- Use colour to separate different ideas – this will help your mind separate ideas where it is necessary, and helps visualisation of the metacog for easy recall. Colour also helps to show organisation.
- Use symbols and images where applicable. If a symbol means something to you, and conveys more information than words, use it. Pictures also help you to remember

information.
- Use shapes, circles and boundaries to connect information – these are additional tools to help show the grouping of information.

Use the concept of analytical geometry as your topic and construct a mind map (or metacog) containing all the information (including terminology, definitions, formulae and examples) that you know about the topic of analytical geometry.

Possible questions to ask the learner on completion of their metacog:
- Briefly explain to me what the mathematics topic of analytical geometry entails.
- Identify and explain the distance formula, the derivation and use thereof for me on your metacog.
- How does the calculation of gradient in analytical geometry differ (or not) from the approach used to calculate gradient in working with functions?

A suggested simple rubric for marking a metacog:

Task	**Competent (2 Marks)**	**Still Developing (1 Mark)**	**Not Yet Developed (0 Marks)**
Completion in Time Limit			
Main Headings			
Correct Theory (Formulae, Definitions, Terminology etc.)			
Explanation			
“Readability”			

10 marks for the questions, which are marked using the following scale:

0 - no attempt or a totally incorrect attempt has been made
1 - a correct attempt was made, but the learner did not get the correct answer
2 - a correct attempt was made and the answer is correct

Investigations

Investigations consist of open-ended questions that initiate and expand thought processes. Acquiring and developing problem-solving skills are an essential part of doing investigations.

It is suggested that 2 – 3 hours be allowed for this task. During the first 30 – 45 minutes learners could be encouraged to talk about the problem, clarify points of confusion, and discuss initial conjectures with others. The final written-up version should be done individually though and should be approximately four pages.

Assessing investigations may include feedback/ presentations from groups or individuals on the results keeping the following in mind:

- following of a logical sequence in solving the problems
- pre-knowledge required to solve the problem
- correct usage of mathematical language and notation
- purposefulness of solution
- quality of the written and oral presentation

Some examples of suggested marking rubrics are included on the next few pages, followed by a selection of topics for possible investigations.

The following guidelines should be provided to learners before they begin an investigation:

General Instructions Provided to Learners

- You may choose any one of the projects/investigations given (see model question on investigations)
- You should follow the instructions that accompany each task as these describe the way in which the final product must be presented.
- You may discuss the problem in groups to clarify issues, but each individual must write-up their own version.
- Copying from fellow learners will cause the task to be disqualified.
- Your educator is a resource to you, and though they will not provide you with answers / solutions, they may be approached for hints.

The Presentation

The investigation is to be handed in on the due date, indicated to you by your educator. It should have as a minimum:

- A description of the problem.
- A discussion of the way you set about dealing with the problem.
- A description of the final result with an appropriate justification of its validity.
- Some personal reflections that include mathematical or other lessons learnt, as well as the feelings experienced whilst engaging in the problem.
- The written-up version should be attractively and neatly presented on about **four** A4 pages.
- Whilst the use of technology is encouraged in the presentation, the mathematical content and processes must remain the major focus.

Below are some examples of possible rubrics to use when marking investigations:

Example 1:

Level of Performance	Criteria
4	• Contains a complete response. • Clear, coherent, unambiguous and elegant explanation. • Includes clear and simple diagrams where appropriate. • Shows understanding of the question’s mathematical ideas and processes.

	• Identifies all the important elements of the question. • Includes examples and counter examples. • Gives strong supporting arguments. • Goes beyond the requirements of the problem.
3	• Contains a complete response. • Explanation less elegant, less complete. • Shows understanding of the question's mathematical ideas and processes. • Identifies all the important elements of the question. • Does not go beyond the requirements of the problem.
2	• Contains an incomplete response. • Explanation is not logical and clear. • Shows some understanding of the question's mathematical ideas and processes. • Identifies some of the important elements of the question. • Presents arguments, but incomplete. • Includes diagrams, but inappropriate or unclear.
1	• Contains an incomplete response. • Omits significant parts or all of the question and response. • Contains major errors. • Uses inappropriate strategies.
0	• No visible response or attempt

Orals

An oral assessment involves the learner explaining to the class as a whole, a group or the educator his or her understanding of a concept, a problem or answering specific questions. The focus here is on the correct use of mathematical language by the learner and the conciseness and logical progression of their explanation as well as their communication skills.

Orals can be done in a number of ways:

- A learner explains the solution of a homework problem chosen by the educator.
- The educator asks the learner a specific question or set of questions to ascertain that the learner understands, and assesses the learner on their explanation.
- The educator observes a group of learners interacting and assesses the learners on their contributions and explanations within the group.
- A group is given a mark as a whole, according to the answer given to a question by any member of a group.

An example of a marking rubric for an oral:

1 - the learner has understood the question and attempts to answer it
2 - the learner uses correct mathematical language
2 - the explanation of the learner follows a logical progression
2 - the learner's explanation is concise and accurate
2 - the learner shows an understanding of the concept being explained
1 - the learner demonstrates good communication skills

Maximum mark = 10

An example of a peer-assessment rubric for an oral:

My name: ________________________

Name of person I am assessing: _______________________

Criteria	**Mark awarded**	**Maximum Mark**
Correct Answer		2
Clarity of Explanation		3
Correctness of Explanation		3
Evidence of Understanding		2
Total		10

Chapter Contexts

Functions and Graphs

Functions form a core part of learners' mathematical understanding and reasoning processes in algebra. This is also an excellent opportunity for contextual mathematical modelling questions.

Number Patterns

Much of mathematics revolves around the identification of patterns.

Finance, Growth and Decay

The mathematics of finance is very relevant to daily and long-term financial decisions learners will need to take in terms of investing, taking loans, saving and understanding exchange rates and their influence more globally.

Algebra

Algebra provides the basis for mathematics learners to move from numerical calculations to generalising operations, simplifying expressions, solving equations and using graphs and inequalities in solving contextual problems.

Products and Factors

Being able to multiply out and factorise are core skills in the process of simplifying expressions and solving equations in mathematics.

Equations and Inequalities

If learners are to later work competently with functions and the graphing and interpretation thereof, their understanding and skills in solving equations and inequalities will need to be developed.

Estimating Surds

Estimation is an extremely important component within mathematics. It allows learners to work with a calculator or present possible solutions while still being able to gauge how accurate and realistic their answers may be. This is relevant for other subjects too. For example, a learner working in biology may need to do a calculation to find the size of the average human kidney. An erroneous interpretation or calculation may result in an answer of 900 m. Without estimation skills, the learner may not query the possibility of such an answer and consider that it should rather be 9 cm. Estimating surds facilitates the further development of this skill of estimation.

Exponentials

Exponential notation is a central part of mathematics in numerical calculations as well as algebraic reasoning and simplification. It is also a necessary component for learners to understand and appreciate certain financial concepts such as compound interest and growth and decay.

Irrational Numbers & Rounding Off

Identifying irrational numbers and knowing their estimated position on a number line or graph is an important part of any mathematical calculations and processes that move beyond the basic number system of whole numbers and integers. Rounding off irrational numbers (such as the value of π) when needed allows mathematics learners to work more efficiently with numbers that would otherwise be difficult to "pin down", use and comprehend.

Rational Numbers

Once learners have grasped the basic number system of whole numbers and integers, it is vital that their understanding of the numbers between integers is also expanded. This incorporates their dealing with fractions, decimals and surds which form a central part of most mathematical calculations in real-life contextual issues.

Differential Calculus: Average Gradient

The central aspect of rate of change to differential calculus is a basis to further understanding of limits, gradients and calculations and formulae necessary for work in engineering fields, e.g. designing roads, bridges etc.

Probability

This topic is helpful in developing good logical reasoning in learners and for educating them in terms of real-life issues such as gambling and the possible pitfalls thereof.

Euclidean Geometry and Measurement

The thinking processes and mathematical skills of proving conjectures and identifying false conjectures is more the relevance here than the actual content studied. The surface area and volume content studied in real-life contexts of designing kitchens, tiling and painting rooms, designing packages, etc. is relevant to the current and future lives of learners.

Trigonometry

Trigonometry has several uses within society, including within navigation, music, geographical locations and building design and construction.

Analytical Geometry

This section provides a further application point for learners' algebraic and trigonometric interaction with the Cartesian plane. Artists and design and layout industries often draw on the content and thought processes of this mathematical topic.

Statistics

Citizens are daily confronted with interpreting data presented from the media. Often this data may be biased or misrepresented within a certain context. In any type of research, data collection and handling is a core feature. This topic also educates learners to become more socially and politically educated with regards to the media.

On the Web, Everyone can be a Scientist

Did you know that you can fold protein molecules, hunt for new planets around distant suns or simulate how malaria spreads in Africa, all from an ordinary PC or laptop connected to the Internet? And you don't need to be a certified scientist to do this. In fact some of the most talented contributors are teenagers. The reason this is possible is that scientists are learning how to turn simple scientific tasks into competitive online games.

This is the story of how a simple idea of sharing scientific challenges on the Web turned into a global trend, called citizen cyberscience. And how you can be a scientist on the Web, too.

Looking for Little Green Men

A long time ago, in 1999, when the World Wide Web was barely ten years old and no one had heard of Google, Facebook or Twitter, a researcher at the University of California at Berkeley, David Anderson, launched an online project called SETI@home. SETI stands for Search for Extraterrestrial Intelligence. Looking for life in outer space.

Although this sounds like science fiction, it is a real and quite reasonable scientific project. The idea is simple enough. If there are aliens out there on other planets, and they are as smart or even smarter than us, then they almost certainly have invented the radio already. So if we listen very carefully for radio signals from outer space, we may pick up the faint signals of intelligent life.
Exactly what radio broadcasts aliens would produce is a matter of some debate. But the idea is that if they do, it would sound quite different from the normal hiss of background radio noise produced by stars and galaxies. So if you search long enough and hard enough, maybe you'll find a sign of life.

It was clear to David and his colleagues that the search was going to require a lot of computers. More than scientists could afford. So he wrote a simple computer program which broke the problem down into smaller parts, sending bits of radio data collected by a giant radio-telescope to volunteers around the world. The volunteers agreed to download a programme onto their home computers that would sift through the bit of data they received, looking for signals of life, and send back a short summary of the result to a central server in California.

The biggest surprise of this project was not that they discovered a message from outer space. In fact, after over a decade of searching, no sign of extraterrestrial life has been found, although there are still vast regions of space that have not been looked at. The biggest surprise was the number of people willing to help such an endeavour. Over a million people have downloaded the software, making the total computing power of SETI@home rival that of even the biggest supercomputers in the world.

David was deeply impressed by the enthusiasm of people to help this project. And he realized that searching for aliens was probably not the only task that people would be willing to help with by using the spare time on their computers. So he set about building a software platform that would allow many other scientists to set up similar projects. You can read more about this platform, called BOINC, and the many different kinds of volunteer computing projects it supports today, at http://boinc.berkeley.edu/ .

There's something for everyone, from searching for new prime numbers (PrimeGrid) to simulating the future of the Earth's climate (ClimatePrediction.net). One of the projects, MalariaControl.net, involved researchers from the University of Cape Town as well as from universities in Mali and Senegal.

The other neat feature of BOINC is that it lets people who share a common interest in a scientific topic share their passion, and learn from each other. BOINC even supports teams – groups of people who put their computer power together, in a virtual way on the Web, to get a higher score than their rivals. So BOINC is a bit like Facebook and World of Warcraft combined – part social network, part online multiplayer game.

Here's a thought: spend some time searching around BOINC for a project you'd like to participate in, or tell your class about.

You are a Computer, too

Before computers were machines, they were people. Vast rooms full of hundreds of government employees used to calculate the sort of mathematical tables that a laptop can produce nowadays in a fraction of a second. They used to do those calculations laboriously, by hand. And because it was easy to make mistakes, a lot of the effort was involved in double-checking the work done by others.

Well, that was a long time ago. Since electronic computers emerged over 50 years ago, there has been no need to assemble large groups of humans to do boring, repetitive mathematical tasks. Silicon chips can solve those problems today far faster and more accurately. But there are still some mathematical problems where the human brain excels.

Volunteer computing is a good name for what BOINC does: it enables volunteers to contribute computing power of their PCs and laptops. But in recent years, a new trend has emerged in citizen cyberscience that is best described as volunteer thinking. Here the computers are replaced by brains, connected via the Web through an interface called eyes. Because for some complex problems – especially those that involve recognizing complex patterns or three-dimensional objects – the human brain is still a lot quicker and more accurate than a computer.

Volunteer thinking projects come in many shapes and sizes. For example, you can help to classify millions of images of distant galaxies (GalaxyZoo), or digitize hand-written information associated with museum archive data of various plant species (Herbaria@home). This is laborious work, which if left to experts would take years or decades to complete. But thanks to the Web, it's possible to distribute images so that hundreds of thousands of people can contribute to the search. Not only is there strength in numbers, there is accuracy, too. Because by using a technique called validation – which does the same sort of double-checking that used to be done by humans making mathematical tables – it is possible to practically eliminate the effects of human error. This is true even though each volunteer may make quite a few mistakes. So projects like Planet Hunters have already helped astronomers pinpoint new planets circling distant stars. The game FoldIt invites people to compete in folding protein molecules via a simple mouse-driven interface. By finding the most likely way a protein will fold, volunteers can help understand illnesses like Alzheimer's disease, that depend on how proteins fold.

Volunteer thinking is exciting. But perhaps even more ambitious is the emerging idea of volunteer sensing: using your laptop or even your mobile phone to collect data – sounds, images, text you type in – from any point on the planet, helping scientists to create global networks of sensors that can pick up the first signs of an outbreak of a new disease (EpiCollect), or the initial tremors associated with an earthquake (QuakeCatcher.net), or the noise levels around a new airport (NoiseTube).

There are about a billion PCs and laptops on the planet, but already 5 billion mobile phones. The rapid advance of computing technology, where the power of a ten-year old PC can easily be packed into a smart phone today, means that citizen cyberscience has a bright future in mobile phones. And this means that more and more of the world's population can be part of citizen cyberscience projects. Today there are probably a few million participants in a few hundred citizen cyberscience initiatives. But there will soon be seven billion brains on the planet. That is a lot of potential citizen cyberscientists.

You can explore much more about citizen cyberscience on the Web. There's a great list of all sorts of projects, with brief summaries of their objectives, at http://distributedcomputing.info/. BBC Radio 4 produced a short series on citizen science http://www.bbc.co.uk/radio4/science/citizenscience.shtml and you can subscribe to a newsletter about the latest trends in this field at http://scienceforcitizens.net/ . The Citizen Cyberscience Centre, www.citizencyberscience.net which is sponsored by the South African Shuttleworth Foundation, is promoting citizen cyberscience in Africa and other developing regions.

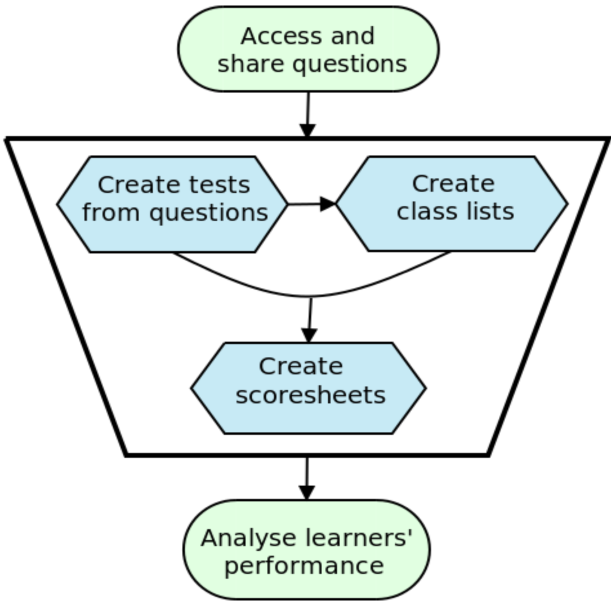
FullMarks User Guide

FullMarks can be accessed at: http://www.fullmarks.org.za/.

Siyavula offers an open online assessment bank called FullMarks, for the sharing and accessing of curriculum-aligned test and exam questions with answers. This site enables educators to quickly set tests and exam papers, by selecting items from the library and adding them to their test. Educators can then download their separate test and memo which is ready for printing. FullMarks further offers educators the option of capturing their learners' marks in order to view a selection of diagnostic reports on their performance.

To begin, you need to a create a free account by clicking on “sign up now” on the landing page. There is one piece of administration you need to do to get started properly: when you log in for the first time, click on your name on the top right. It will take you to your personal settings. You need to select Shuttleworth Foundation as your metadata organisation to see the curriculum topics.

What Can I do in FullMarks?



How do I do Each of These?

Access and Share Questions

Sharing questions: use either the online editor or OpenOffice template which can be downloaded from the website (Browse questions → Contribute questions → Import questions). Take your test/worksheet/other question source. Break it up into the smallest sized individual questions that make sense, and use the template style guide to style your page according to question/answer. Upload these questions or type them up in the online editor (Browse questions → Contribute questions → Add questions). Do not include overall question numbering but do include sub numbering if needed (e.g. 1a, 2c, etc.). Insert the mark and time allocation, tag questions according to grade, subject i.e. a description of the question, and then select the topics from the topic tree . Finalise questions so that they can be used in tests and accessed by other FullMarks users.

Accessing questions: there are three ways to access questions in the database. Click on "Browse questions" → click on the arrow to the left of the grade, which opens out the subjects → keep clicking on the arrows to open the learning outcomes **or,** following the same process, instead of clicking on the arrow, click on the grade → now you can browse the full database of questions for all the subjects in that grade **or,** from the landing page, click on "Browse questions" → below the banner image click on "Find Questions" → search by topics, author (if you know a contributor), text or keywords e.g. Gr10 mathematics functions and graphs.

Create Tests from Questions

So, you have all these bits of tests (i.e. many questions!), but what you really want is the actual test. How do you do this? Well, you can simply click "add to test" on any question and then click on the "Tests" tab at the top right of the page, and follow the simple instructions. Alternatively you can create a test by starting with clicking on that same "Tests" tab, and add questions to your test that way. Once done, simply print off the PDF file of the questions and the file for the memo. Issue your test, collect them once complete, and mark them.

Create Class Lists

But now you are asking, how can I keep track of my classes? Is Johnny Brown in class A or B? Well, you can make a class list by clicking on the "Class lists" tab at the top right of the page, and either import a CSV file, or manually enter the relevant information for each class. Now you can issue tests to your classes, and have a class list for each class. And what about capturing their marks?

Create Scoresheets

For each test you can create a scoresheet. Select the "Tests" tab at the top right, click on "Marks" below the banner image, and select the test and follow the instructions to input their marks. You can then export these as a CSV file for use in spreadsheets.

Analyse Learners' Performance

And finally, you can print out reports of class performance. Click on "Reports" at the top right of the page, which opens various reports you can view. There are reports to see class performance, learner performance, class performance per topic, class performance per question, learner strengths and weaknesses, and learner progress.

So now you know how FullMarks works, we encourage you to make use of its simple functionality, and let it help you save time setting tests and analysing learner marks!

Rich Media

General Resources

Science education is about more than physics, chemistry and mathematics... It's about learning to think and to solve problems, which are valuable skills that can be applied through all spheres of life. Teaching these skills to our next generation is crucial in the current global environment where methodologies, technology and tools are rapidly evolving. Education should benefit from these fast moving developments. In our simplified model there are three layers to how technology can significantly influence your teaching and teaching environment.

First Layer: Educator Collaboration

There are many tools that help educators collaborate more effectively. We know that communities of practice are powerful tools for the refinement of methodology, content and knowledge as well as superb for providing support to educators. One of the challenges facing community formation is the time and space to have sufficient meetings to build real communities and exchanging practices, content and learnings effectively. Technology allows us to streamline this very effectively by transcending space and time. It is now possible to collaborate over large distances (transcending space) and when it is most appropriate for each individual (transcending time) by working virtually (email, mobile, online etc.).

Our textbooks have been uploaded in their entirety to the Connexions website (http://cnx.org/lenses/fhsst), making them easily accessible and adaptable, as they are under an open licence, stored in an open format, based on an open standard, on an open-source platform, for free, where everyone can produce their own books. Our textbooks are released under an open copyright license - CC-BY. This Creative Commons By Attribution Licence allows others to legally distribute, remix, tweak, and build upon our work, even commercially, as long as they credit us for the original creation. With them being available on the Connexions website and due to the open copyright licence, learners and educators are able to download, copy, share and distribute our books legally at no cost. It also gives educators the freedom to edit, adapt, translate and contextualise them, to better suit their teaching needs.

Connexions is a tool where individuals can share, but more importantly communities can form around the collaborative, online development of resources. Your community of educators can therefore:

- form an online workgroup around the textbook;
- make your own copy of the textbook;
- edit sections of your own copy;
- add your own content into the book or replace existing content with your content;
- use other content that has been shared on the platform in your own book;
- create your own notes / textbook / course material as a community.

Educators often want to share assessment items as this helps reduce workload, increase variety and improve quality. Currently all the solutions to the exercises contained in the textbooks have been uploaded onto our free and open online assessment bank called FullMarks (http://www.fullmarks.org.za/), with each exercise having a shortcode link to its solution on FullMarks. To access the solution simply go to http://www.siyavula.com, enter the shortcode, and you will be redirected to the solution on FullMarks.

FullMarks is similar to Connexions but is focused on the sharing of assessment items. FullMarks contains a selection of test and exam questions with solutions, openly shared by educators. Educators can further search and browse the database by subject and grade and add relevant items to a test. The website automatically generates a test or exam paper with the corresponding memorandum for download.

By uploading all the end-of-chapter exercises and solutions to the open assessment bank, the larger community of educators in South Africa are provided with a wide selection of items to use in setting their tests and exams. More details about the use of FullMarks as a collaboration tool are included in the FullMarks section.

Second Layer: Classroom Engagement

In spite of the impressive array of rich media open educational resources available freely online, such as videos, simulations, exercises and presentations, only a small number of educators actively make use of them. Our investigations revealed that the overwhelming quantity, the predominant international context, and difficulty in correctly aligning them with the local curriculum level acts as deterrents. The opportunity here is that, if used correctly, they can make the classroom environment more engaging.

Presentations can be a first step to bringing material to life in ways that are more compelling than are possible with just a blackboard and chalk. There are opportunities to:

- create more graphical representations of the content;
- control timing of presented content more effectively;
- allow learners to relive the lesson later if constructed well;
- supplement the slides with notes for later use;
- embed key assessment items in advance to promote discussion; and
- embed other rich media like videos.

Videos have been shown to be potentially both engaging and effective. They provide opportunities to:

- present an alternative explanation;
- challenge misconceptions without challenging an individual in the class; and
- show an environment or experiment that cannot be replicated in the class which could be far away, too expensive or too dangerous.

Simulations are also very useful and can allow learners to:

- have increased freedom to explore, rather than reproducing a fixed experiment or process;
- explore expensive or dangerous environments more effectively; and
- overcome implicit misconceptions.

We realised the opportunity for embedding a selection of rich media resources such as presentations, simulations, videos and links into the online version of the FHSST books at the relevant sections. This will not only present them with a selection of locally relevant and curriculum aligned resources, but also position these resources within the appropriate grade and section. Links to these online resources are recorded in the print or PDF versions of the books, making them a tour-guide or credible pointer to the world of online rich media available.

Third Layer: Beyond the Classroom

The internet has provided many opportunities for self-learning and participation which were never before possible. There are huge stand-alone archives of videos like the Khan Academy which cover most Mathematics for Grades 1 - 12 and Science topics required in FET. These videos, if not used in class, provide opportunities for the learners to:

- look up content themselves;
- get ahead of class;
- independently revise and consolidate their foundation; and
- explore a subject to see if they find it interesting.

There are also many opportunities for learners to participate in science projects online as real participants. Not just simulations or tutorials but real science so that:

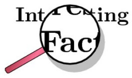
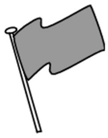
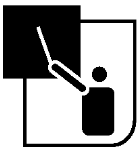
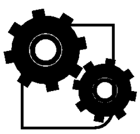
- learners gain an appreciation of how science is changing;
- safely and easily explore subjects that they would never have encountered before university;
- contribute to real science (real international cutting edge science programmes);
- have the possibility of making real discoveries even from their school computer laboratory; and
- find active role models in the world of science.

In our book we've embedded opportunities to help educators and learners take advantage of all these resources, without becoming overwhelmed at all the content that is available online.

Embedded Content

Throughout the books you will see the following icons:

Icon	Description
	**Aside:** Provides additional information about content covered in the chapters, as well as for extensions

	**An interesting fact:** These highlight interesting information relevant to a particular section of the chapter.
	**Definition:** This icon indicates a definition.
	**Exercise:** This indicates worked examples throughout the book.
	**Tip:** Helpful hints and tips appear throughout the book, highlighting important information, things to take note of, and areas where learners must exercise caution.
	**FullMarks:** This icon indicates that shortcodes for FullMarks are present. Enter the shortcode into http://www.siyavula.com, and you will be redirected to the solution on FullMarks, our free and open online assessment bank. FullMarks can be accessed at: http://www.fullmarks.org.za/
	**Presentation:** This icon indicates that presentations are in the chapter. Enter the shortcode into http://www.siyavula.com, and you will be redirected to the presentation shared on SlideShare by educators. SlideShare can be accessed at: http://www.slideshare.net/
	**Simulation:** This icon indicates that simulations are present. Enter the shortcode into http://www.siyavula.com, and you will be redirected to the simulation online. An example is Phet Simulations. The website can be accessed at: http://phet.colorado.edu/
	**Video:** This icon indicates that videos are present. Enter the shortcode into http://www.siyavula.com, and you will be redirected to the video online. An example is the Khan Academy videos. The website can be accessed at: http://www.khanacademy.org/

WWW	**URL:** This icon indicates that shortcodes are present in the chapter and can be entered into http://www.siyavula.com, where you will be redirected to the relevant website.

Blog Posts

General Blogs

- **Teachers Monthly** - Education News and Resources
  - “We eat, breathe and live education! “
  - “Perhaps the most remarkable yet overlooked aspect of the South African teaching community is its enthusiastic, passionate spirit. Every day, thousands of talented, hard-working teachers gain new insight from their work and come up with brilliant, inventive and exciting ideas. Teacher’s Monthly aims to bring teachers closer and help them share knowledge and resources.
  - Our aim is twofold …
- To keep South African teachers updated and informed.
- To give teachers the opportunity to express their views and cultivate their interests.”
- http://www.teachersmonthly.com

- **Head Thoughts** – Personal Reflections of a School Headmaster
  - blog by Arthur Preston
  - “Arthur is currently the headmaster of a growing independent school in Worcester, in the Western Cape province of South Africa. His approach to primary education is progressive and is leading the school through an era of new development and change.”
- http://headthoughts.co.za/

Maths Blogs

- **CEO: Circumspect Education Officer** - Educating The Future
  - blog by Robyn Clark
  - “Mathematics teacher and inspirer.”
- http://clarkformaths.tumblr.com/

- **dy/dan** - Be less helpful
  - blog by Dan Meyer
  - “I'm Dan Meyer. I taught high school math between 2004 and 2010 and I am currently studying at Stanford University on a doctoral fellowship. My specific interests include curriculum design (answering the question, "how we design the ideal learning experience for students?") and teacher education (answering the questions, "how do teachers learn?" and "how do we retain more teachers?" and "how do we teach teachers to teach?").”
- http://blog.mrmeyer.com

- **Without Geometry, Life is Pointless** - Musings on Math, Education, Teaching, and Research
  - blog by Avery
  - “I've been teaching some permutation (or is that combination?) of math and science to third through twelfth graders in private and public schools for 11 years. I'm also pursuing my EdD in education and will be both teaching and conducting research in my classroom this year.”
- http://mathteacherorstudent.blogspot.com/

- **Overthinking my teaching** - The Mathematics I Encounter in Classrooms
  - blog by Christopher Danielson
  - “I think a lot about my math teaching. Perhaps too much. This is my outlet. I hope you find it interesting and that you’ll let me know how it’s going.”
- http://christopherdanielson.wordpress.com

- **A Recursive Process -** Math Teacher Seeking Patterns
  - blog by Dan
  - “I am a High School math teacher in upstate NY. I currently teach Geometry, Computer Programming (Alice and Java), and two half year courses: Applied and Consumer Math. This year brings a new 21st century classroom (still not entirely sure what that entails) and a change over to standards based grades (#sbg).”
- http://dandersod.wordpress.com

- **Think Thank Thunk –** Dealing with the Fear of Being a Boring Teacher
  - blog by Shawn Cornally
  - “I am Mr. Cornally. I desperately want to be a good teacher. I teach Physics, Calculus, Programming, Geology, and Bioethics. Warning: I have problem with using colons. I proof read, albeit poorly.”
- http://101studiostreet.com/wordpress/

Chapter 1 - Review of Past Work

http://cnx.org/content/m38346/latest/?collection=col11306/latest

Solutions

Negative Numbers

1.

a)
$$\begin{aligned}(-5)-(-3) &= (-5)+3\\ &= 3-5\\ &= -2\end{aligned}$$

b)
$$\begin{aligned}(-4)+2 &= 2-4\\ &= -2\end{aligned}$$

c)
$$(-10)\div(-2) = 5$$

d)
$$\begin{aligned}11-(-9) &= 11+9\\ &= 20\end{aligned}$$

e)
$$\begin{aligned}-16-(6) &= -16-6\\ &= -22\end{aligned}$$

f)
$$\begin{aligned}-9\div3\times2 &= -9\div6\\ &= -3\div2\\ &= -\tfrac{3}{2}\end{aligned}$$

g)
$$\begin{aligned}(-1)\times24\div8\times(-3) &= -24\div-24\\ &= 1\end{aligned}$$

h)
$$\begin{aligned}(-2)+(-7) &= -2-7\\ &= -9\end{aligned}$$

i)
$$1-12 = -11$$

j)
$$\begin{aligned}3-64+1 &= 3+1-64\\ &= 4-64\\ &= 60\end{aligned}$$

k)
$$\begin{aligned}-5-5-5 &= -10-5\\ &= -15\end{aligned}$$

l)
$$\begin{aligned}-6+25 &= 25-6\\ &= 19\end{aligned}$$

m)
$$\begin{aligned}-9+8-7+6-5+4-3+2-1 &= -9+(8-7)+(6-5)+(4-3)+(2-1)\\ &= -9+1+1+1+1\\ &= -9+4\\ &= -5\end{aligned}$$

2.

a) positive

b) negative

c) positive

d) negative

e) negative

f) positive

g) positive

h) negative

i) negative

j) positive

Rearranging Equations

1. If $3(2r-5)=27$, then $2r-5=\frac{27}{3}=9$

2. $$\begin{aligned} 0,5x-\frac{8}{2} &= 0,2x+11 \\ 0,3x &= 15 \\ x &= 15\div\frac{3}{10} \\ &= \frac{150}{3} \\ &= 50 \end{aligned}$$

3. $$\begin{aligned} 9-2n &= 3n+6 \\ 3 &= 5n \\ n &= \frac{3}{5} \end{aligned}$$

4. $$\begin{aligned} P &= A+Akt \\ &= A(1+kt) \\ A &= \frac{P}{1+kt} \end{aligned}$$

5. $$\begin{aligned} \frac{1}{ax}+\frac{1}{bx} &= 1 \\ \frac{ax}{ax.bx}+\frac{bx}{ax.bx} &= 1 \\ \frac{ax+bx}{ax.bx} &= 1 \\ \frac{x(a+b)}{a.b.x^2} &= 1 \\ \frac{a+b}{a.b.x} &= 1 \\ \frac{a+b}{a.b} &= x \\ x &= \frac{a+b}{a.b} \end{aligned}$$

Real Numbers

1. a) irrational
   b) rational

   c) integer

   d) rational

   e) irrational

2. Real and rational. $\sqrt{4}=2$ which is an integer and also rational. $\frac{1}{8}$ is rational as it is expressed as division of integers.

Mathematical Symbols

1. a) $x > 1$
   b) $y \leq z$
   c) $a \leq 21$
   d) $21 < p \leq 25$

End of Chapter Exercises

1. Remember order of operation: Brackets, then division, multiplication, addition and subtraction

   a) 6

   b) 34

   c) 36

   d) 4

   e) 20

   f) 20

   g) 9

   h) 15

   i) 61

   j) 54

2. $r = \frac{p - q}{4}$

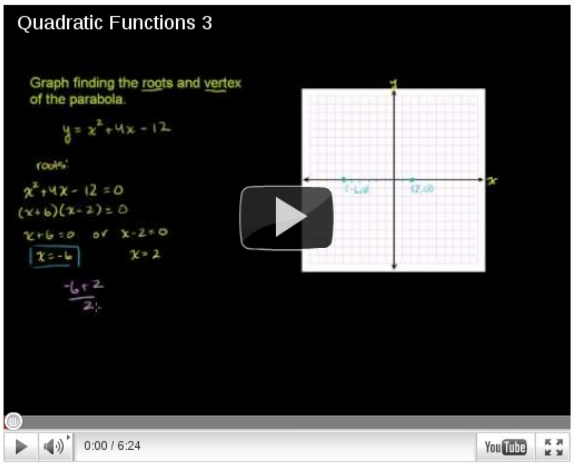
3. $$\begin{aligned} x - 2 &= 3 \quad x - 3 \\ x - 2 &= 3x - 9 \\ 3x - x &= 9 - 2 \\ 2x &= 7 \\ x &= \tfrac{7}{2} = 3{,}5 \end{aligned}$$

Chapter 2 - Functions and Graphs

http://cnx.org/content/m38379/latest/?collection=col11306/latest

Videos

Khan academy video on graphing parabolas - 1

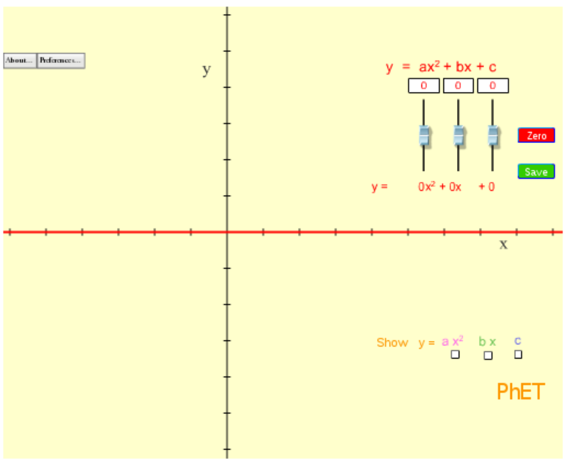


This Khan Academy video on Quadratic Functions 3 covers the concept of graphing a parabola by finding the roots and vertex. This video works through a sample problem and guides learners through the process of graphing a parabola.

This video can be downloaded at: http://www.fhsst.org/lCY

Simulations

Phet simulation for graphing



This Phet Simulation on graphing allows students to see the effect of changing the coefficients of a quadratic equation. This simulation can also be used to help students see the effect of changing the coefficients of a linear equation.

This simulation can be viewed at: http://www.fhsst.org/lCg

A lesson plan for this simulation can be found at: http://www.fhsst.org/lbl

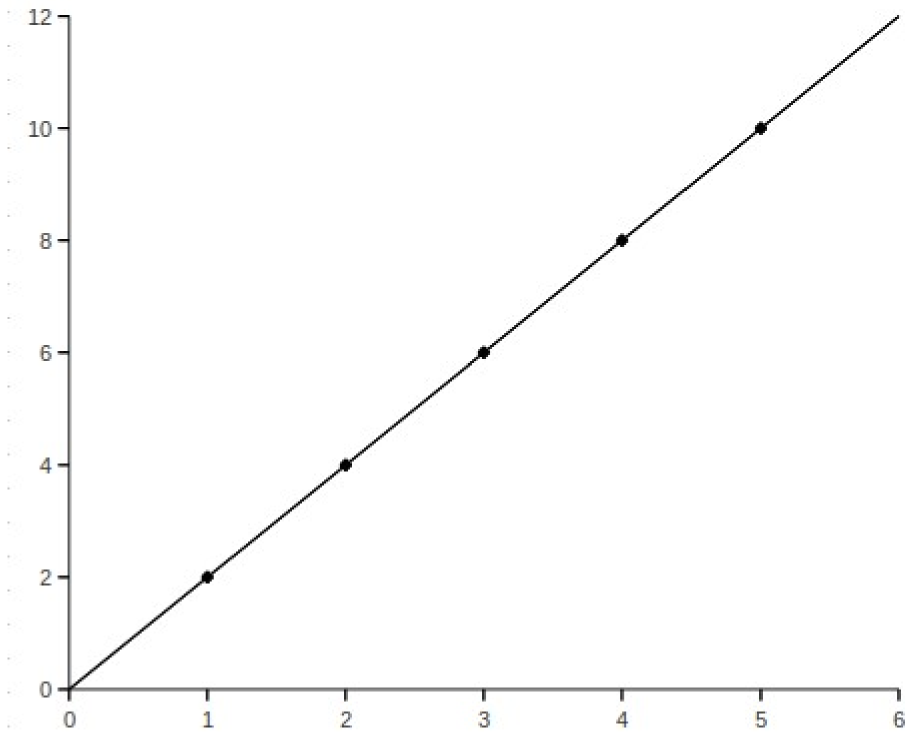
Solutions

Recap

1. For each value of x that we put in, we get the same value for y and so the equation is simply:
   y = x

2. For each value of x that we put in, we get twice the value back and so the equation is:
   y = 2x

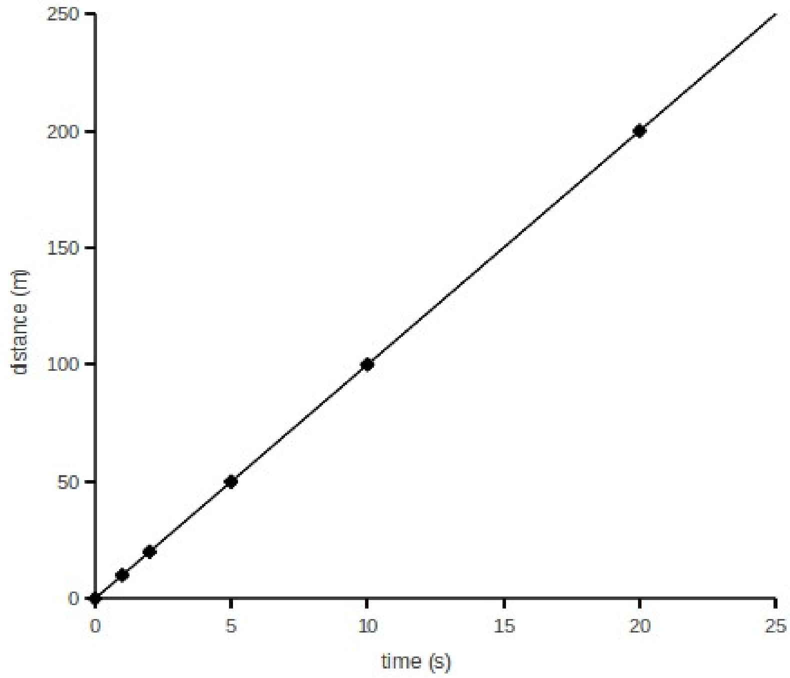
3. For each value of x that we put in, we get 10 times the value back and so the equation is:
   y = 10x

4. 

5. a) $f(t)=t+t^2$
   b) $f(a)=a+a^2$
   c) $f(1)=1+1^2=2$
   d) $f(3)=3+3^2=3+9=12$

6. a) $f(t)+g(t)=t+2t=3t$
   b) $f(a)-g(a)=2a-a=a$
   c) $f(1)+g(2)=2(1)+2=4$
   d) $f(3)+g(s)=2(3)+s=6+s$

7.



Time (s)	012510				20
Distance (m)	010	20	50	100	200

Domain and Range

1. We substitute in the x-values to get the range:
   $f(-3)=2(-3)+5=-1$

   and
   $f(0)=2(0)+5=5$

   So the range is (-1;5)

2. a) The domain is (-3;3)

   b) The range is:

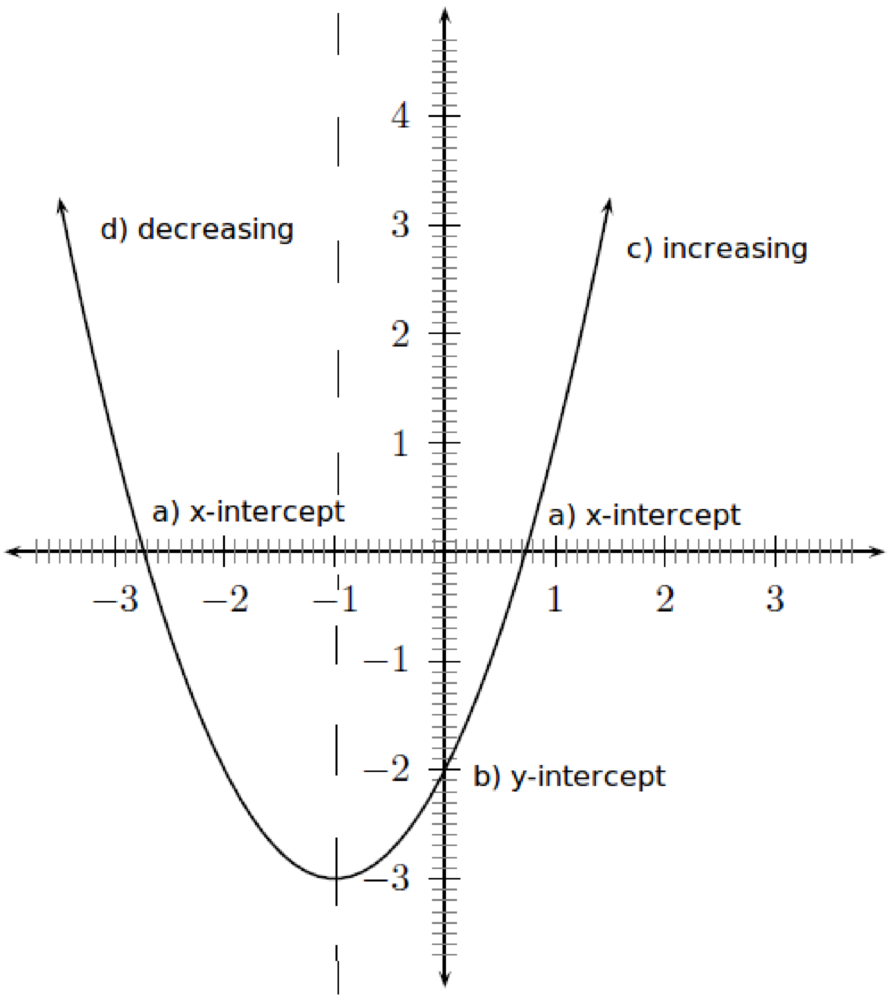
   $f(-3)=-(-3)^2+5=-4$

   $f(3)=-(3)^2+5=-4$

   When x = 0, y = 5

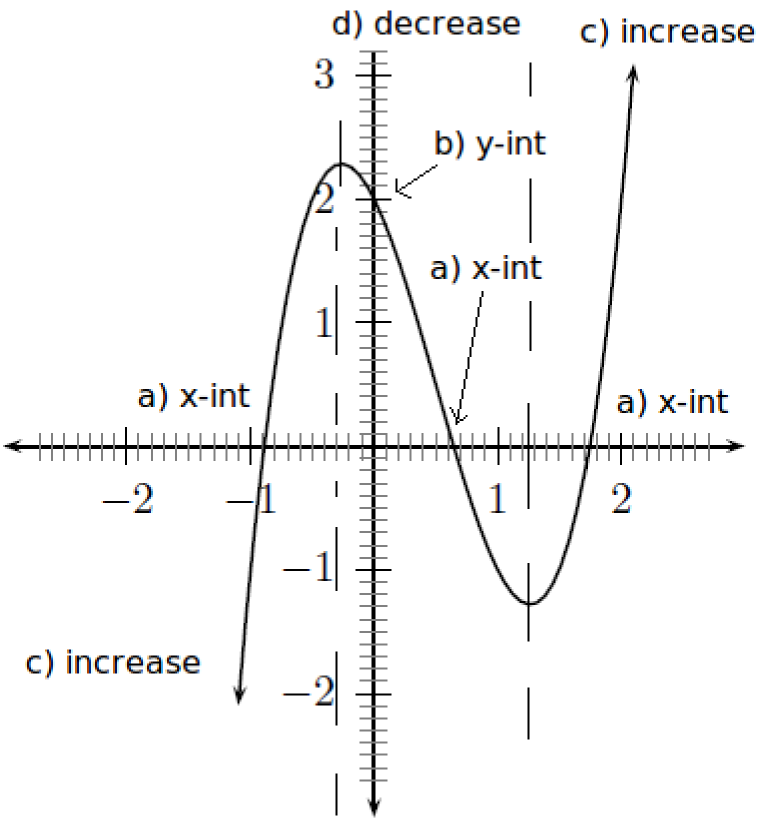
   The range is: (-4;5)

3.



The dotted line on the graph indicates where the graph stops decreasing and starts increasing. On the left side of the line the graph is decreasing. On the right side of the line the graph is increasing.

4.



The graph is increasing on the left side of the first dotted line and increasing again on the right side of the second dotted line. Between the lines the graph is decreasing.

Intercepts

1. a) y-intercept = 0

   b) Set x = 0:

   $y = 0 - 1 = -1$

   So the y-intercept is -1

   c) Set x = 0

   $y = 2(0) - 1 = -1$

   So the y-intercept is -1

   d) Set x = 0

   $y + 1 = 2(0)$

   $y = -1$

   So the y-intercept is -1

2. To find the equation of a straight-line we use:

   $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slope and the y-intercept to find c.

   So the slope is:

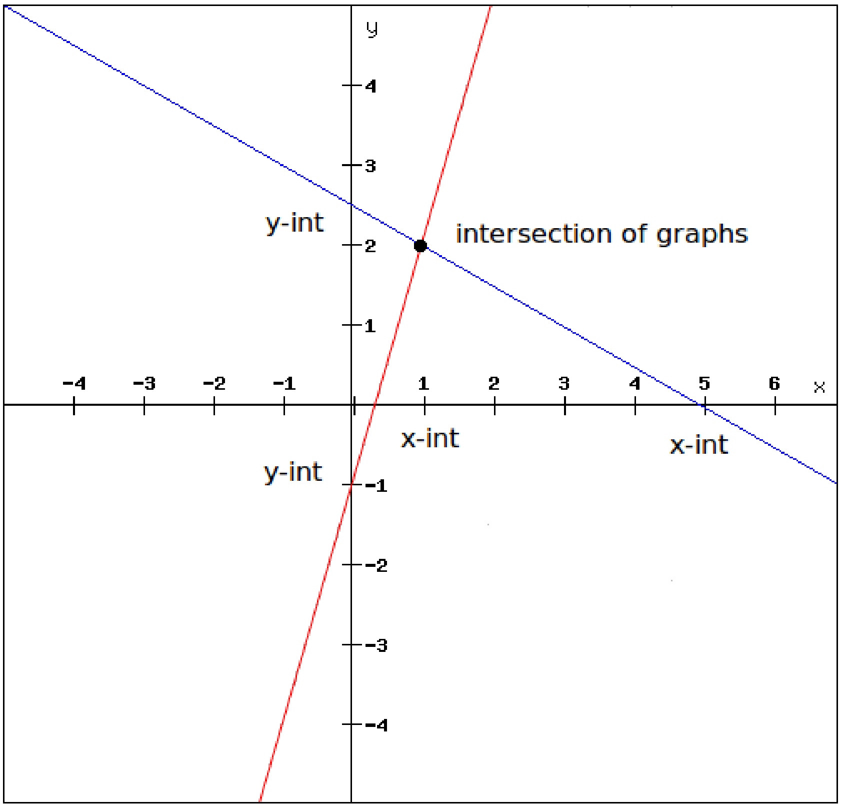
   $m = \frac{0-3}{4-0} = \frac{-3}{4}$

   The y-intercept is 3.

   The equation is:

   $y = \frac{-3}{4}x + 3$

3.


y
4
3
2
1
-1
-2
-3
-4
-4
-3
-2
-1
1
2
3
4
5
6
x
y-int
intersection of graphs
x-int
x-int
y-int

Parabolas

1. Since the square of any number is always positive we get: $x^2 \geq 0$

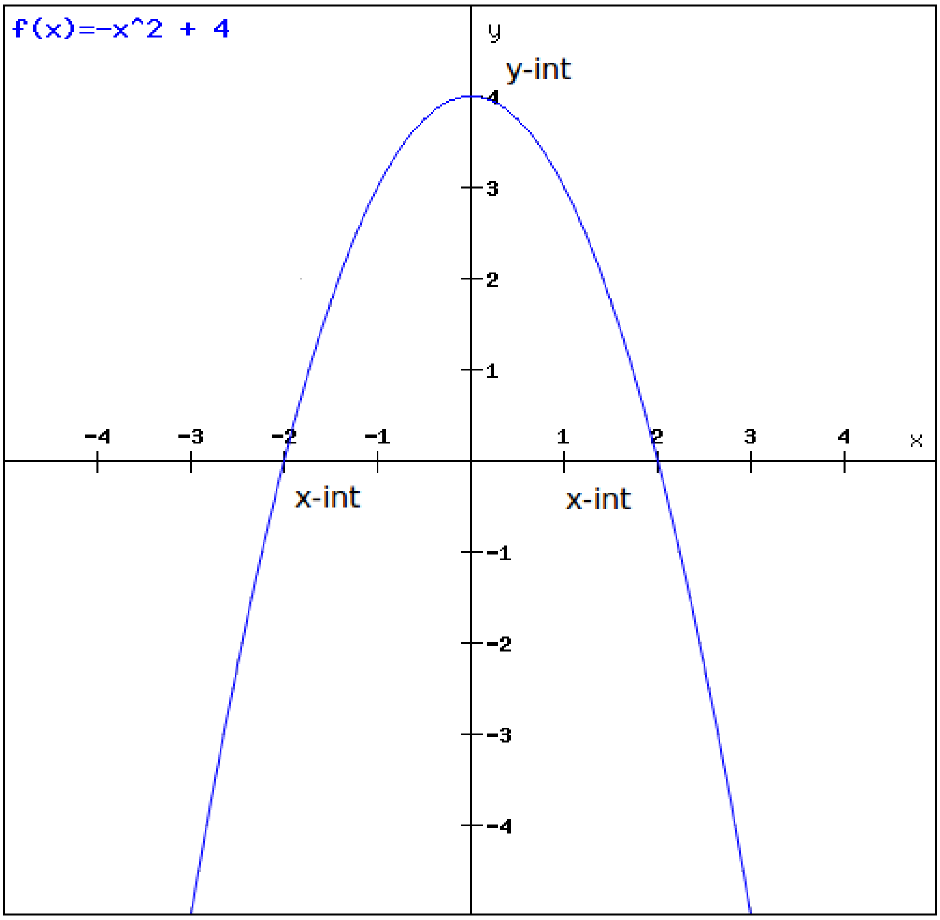
   If we multiply by a (a < 0) then the sign of the inequality changes: $ax^2 \leq 0$

   Adding q to both sides gives: $ax^2 + q \leq q$

   And so $f(x) \leq q$

   This gives the range as:

   $(-\infty, q]$

2. 

3. a) p is the y-intercept and so p = -9.

To find a, we use one of the points on the graph (e.g. (4;7))

$$
\begin{aligned}
y&=ax^2-9\\
7&=a(4^2)-9\\
16&=16a\\
a&=1
\end{aligned}
$$

b) q is the y-intercept and so q = 23

To find b, we use one of the points on the graph (e.g. (4;7))

$$
\begin{aligned}
y&=bx^2+23\\
7&=16b+23\\
-16&=16b\\
b&=-1
\end{aligned}
$$

c) This is the point where g lies above h.

From the graph we see that g lies above h when:

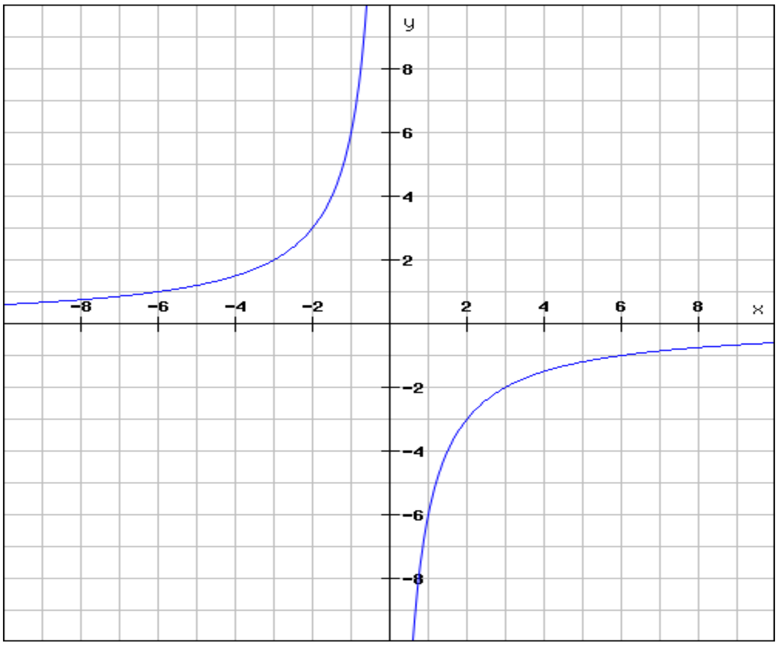
$x \leq -4$ or $x \geq 4$

d) $x \geq 0$

(0;-9) is the turning point.

Graphs

1.



a) We substitute in the x and y values:

$xy=(-2)(3)=-6$

Since this gives us -6, the point (-2,3) does lie on the graph. (i.e. it fulfills the equation)

b) $xy=(-2)(-3)=6$

This does not fulfill the equation xy = -6.

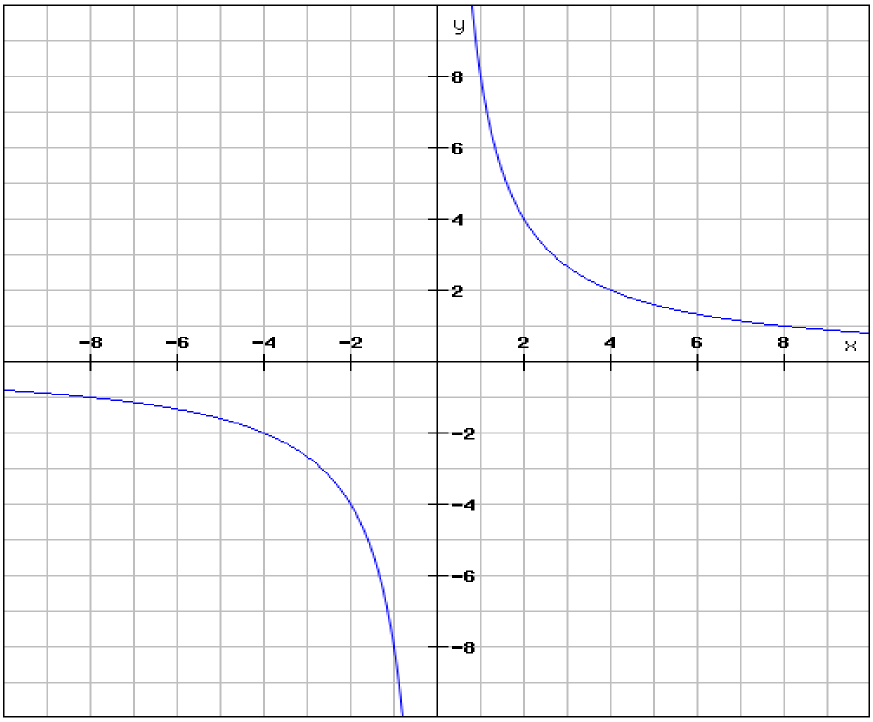
c) Substitute in x = 0,25

$$
\begin{aligned}
0{,}25y&=-6\\
y&=\frac{-6}{\frac{1}{4}}\\
y&=-6\times 4\\
y&=-24
\end{aligned}
$$

d) As the x-values become very large the y-values become very small.

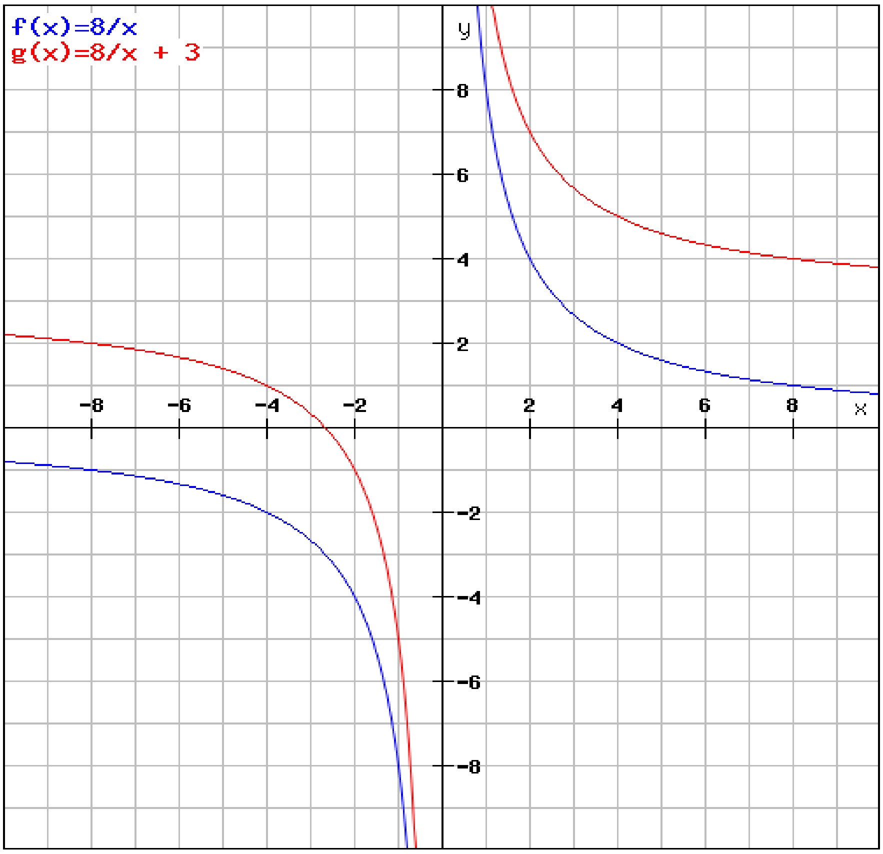
e) (-3;2)

2.



a) It would be shifted 3 units vertically upwards. In other words all the y-values would increase.

b)



Exponential Functions and Graphs

1. a) The x-axis is an asymptote for both graphs, but not an axis of symmetry. Both graphs approach the x-axis but never touch it.

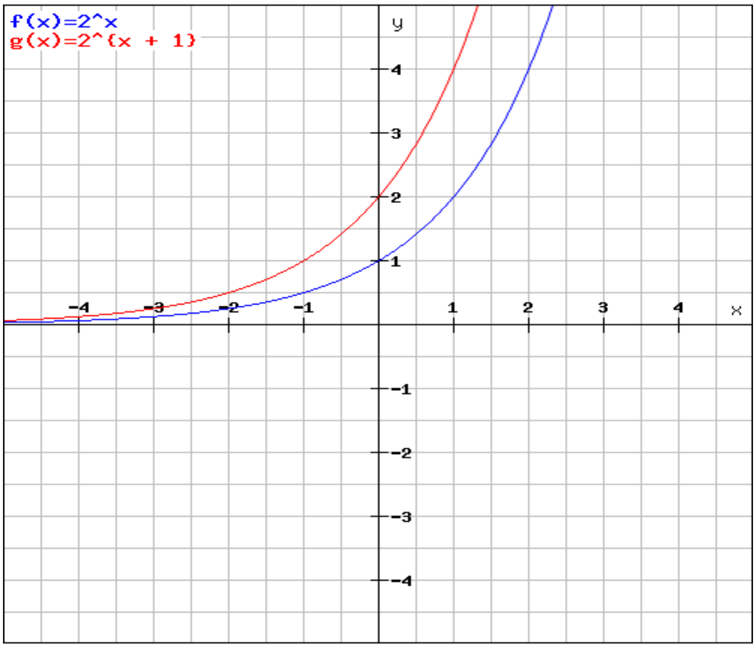
   b) g(x) since $\frac{1}{2}=2^{-1}$ which gives $g(x)=2^{-x}$

   c) This is the point of intersection which is: (0;1)

   Algebraically we get:

   $f(0)=2^0=1$ and $g(0)=2^{-0}=1$.

$$2\times 2^x=2^{x+1}$$



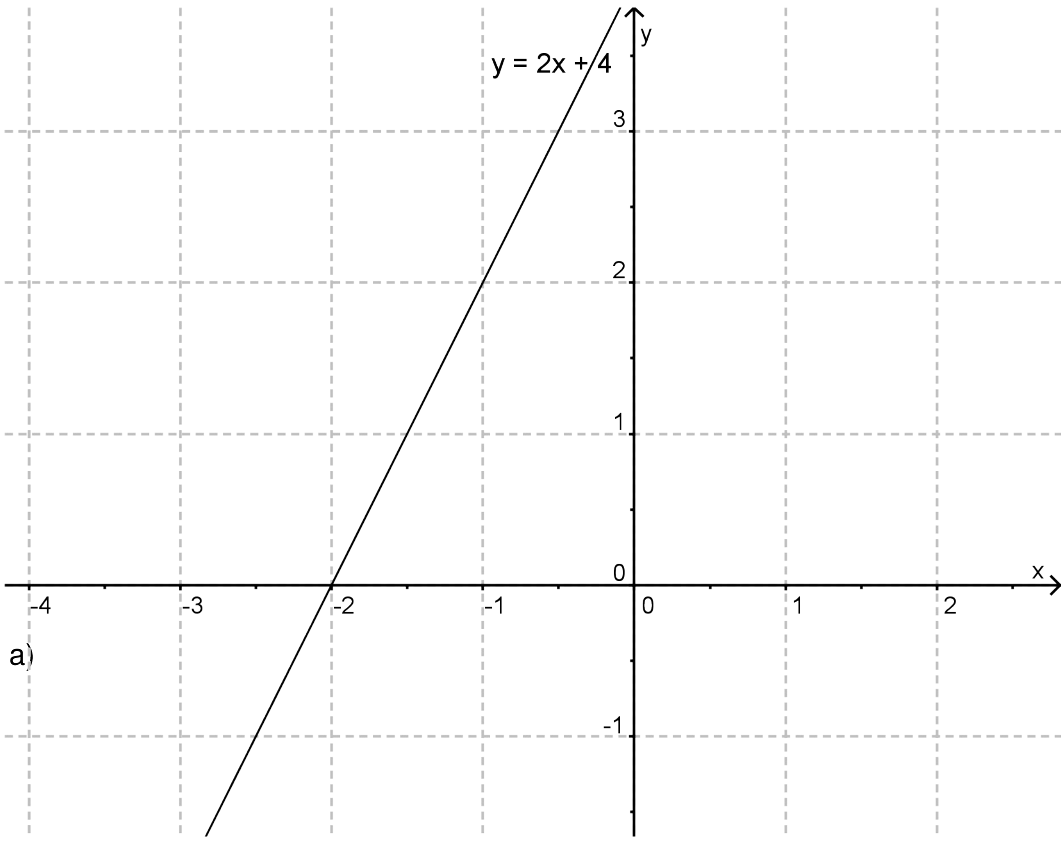
2. a) $f(x)=2^x$

   b) $h(x)=-2^x$

   c) $(-\infty,0)$

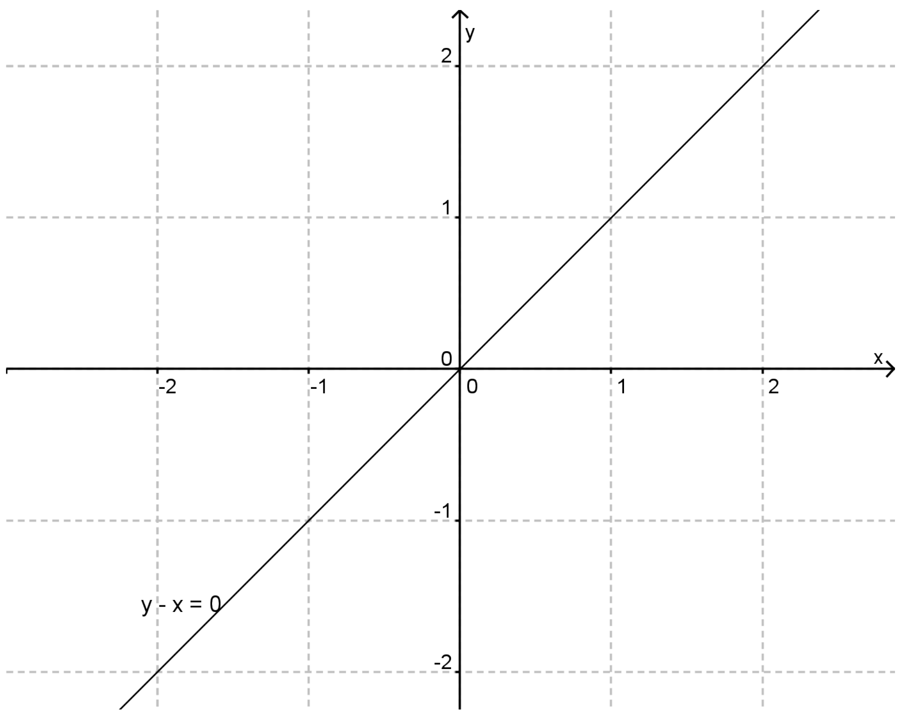
End of Chapter Exercises

1.

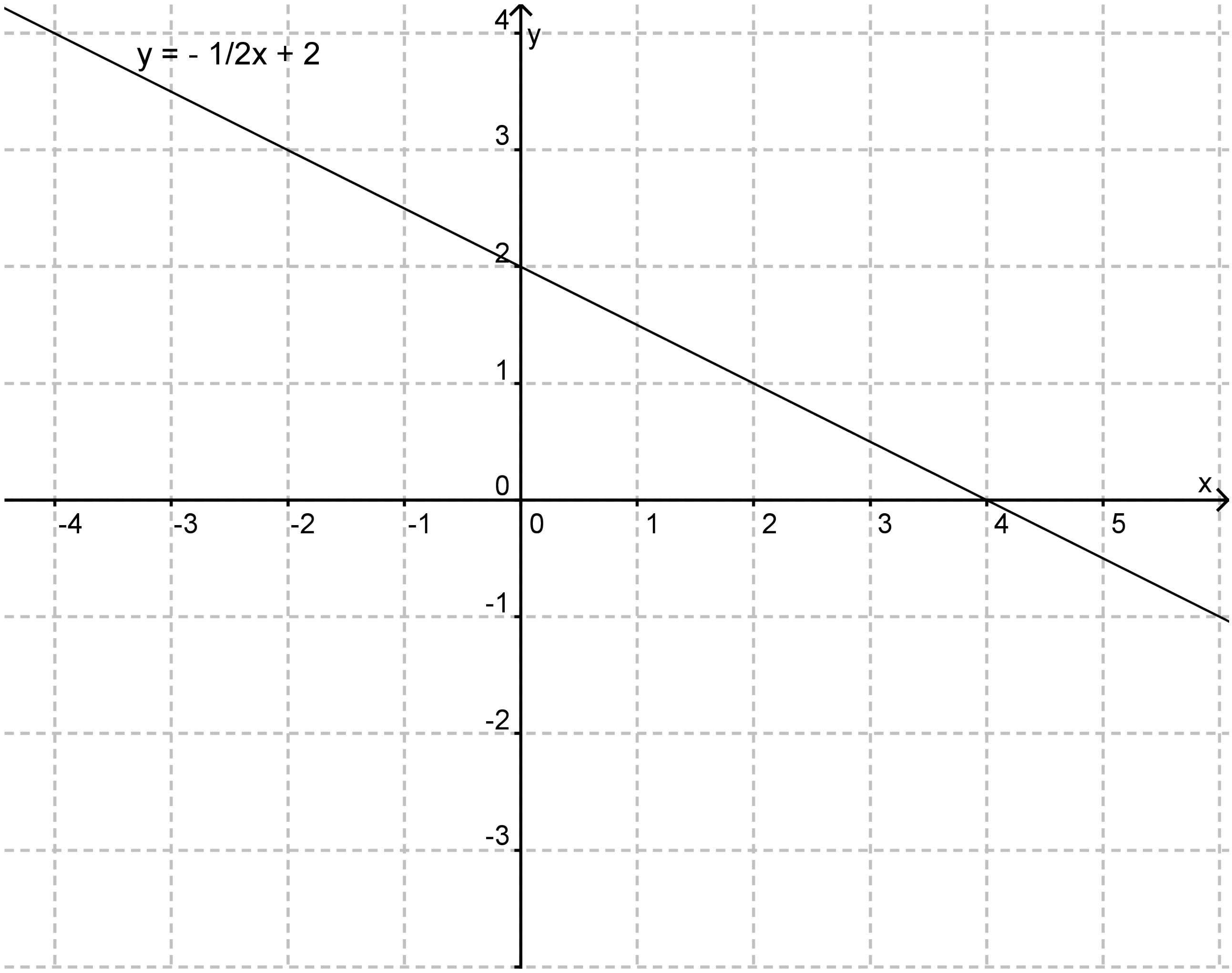


a)

b)

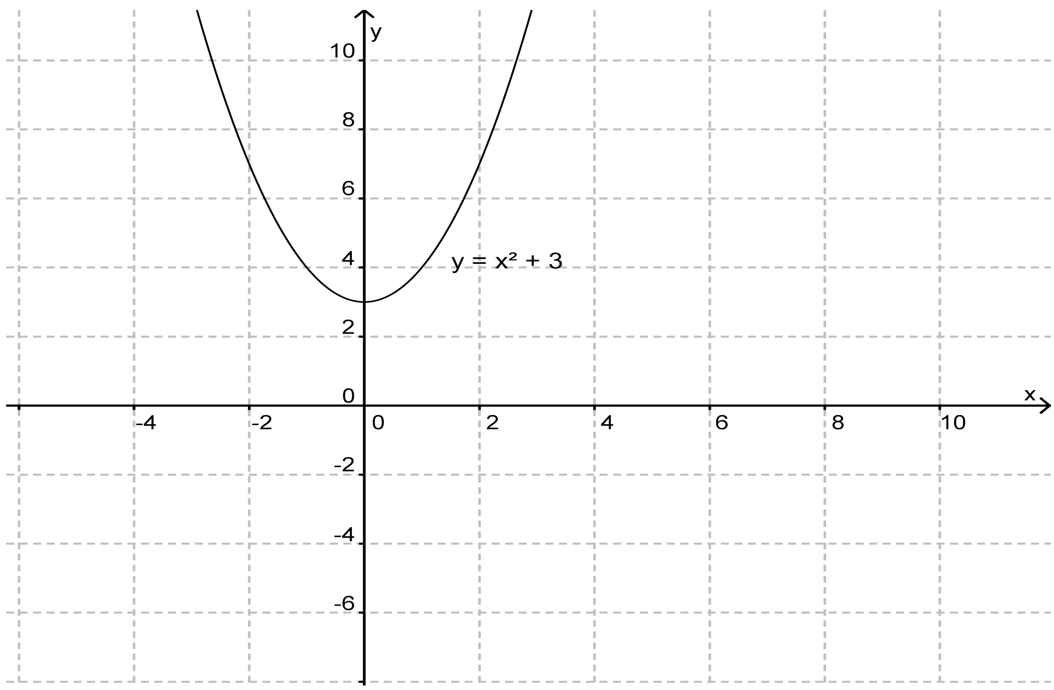


c)



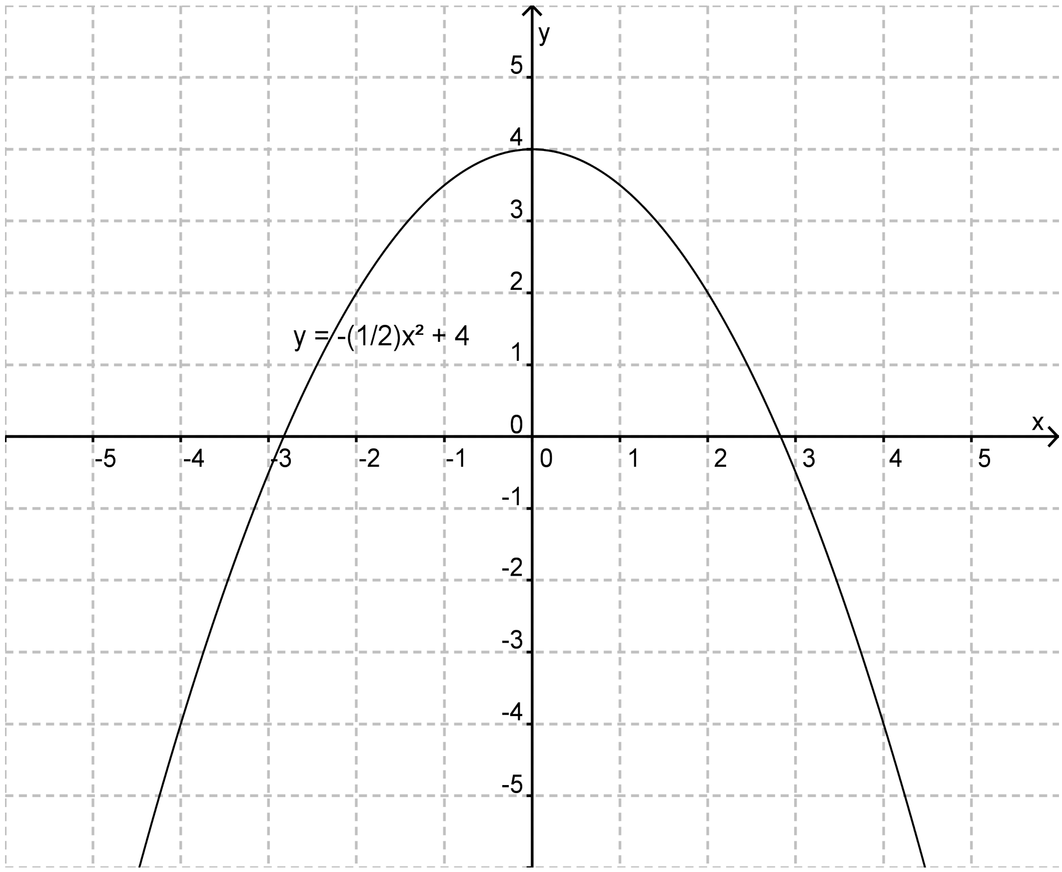
y = - 1/2x + 2
y
x
4
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4
5

2. a)

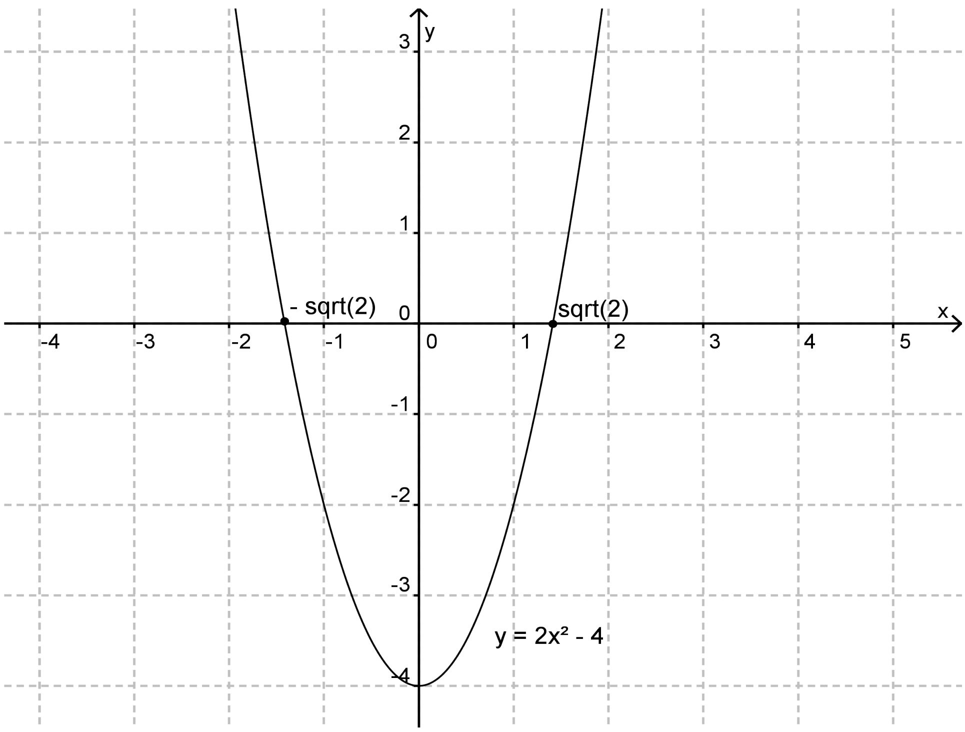


y
x
y = x² + 3
10
8
6
4
2
0
-2
-4
-6
-4
-2
0
2
4
6
8
10

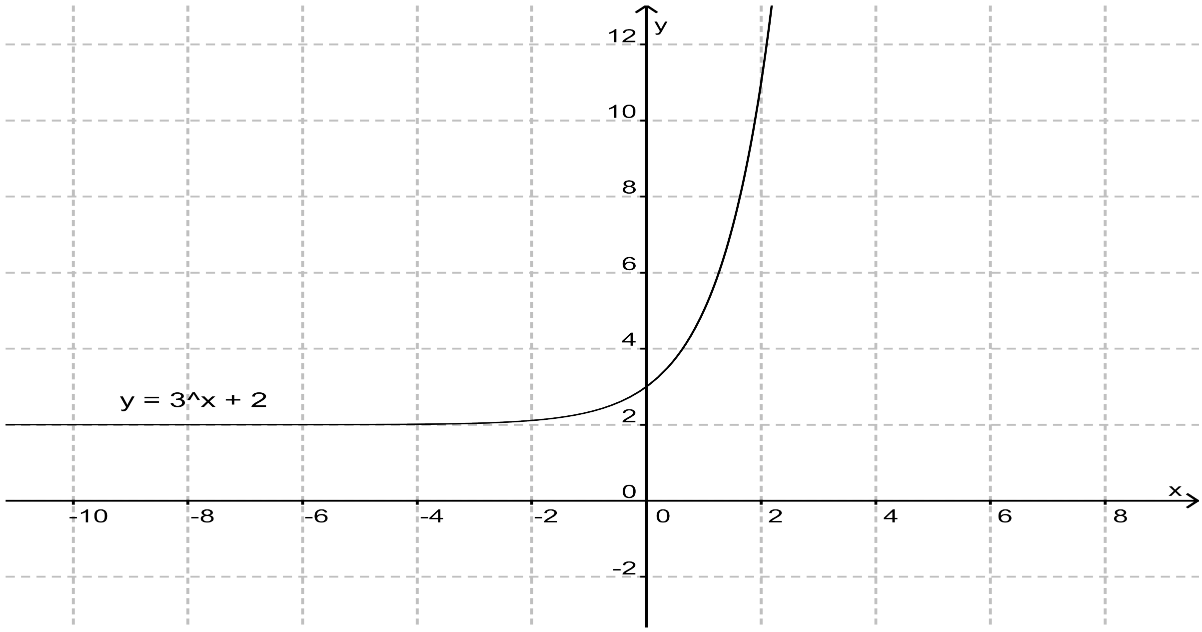
b)



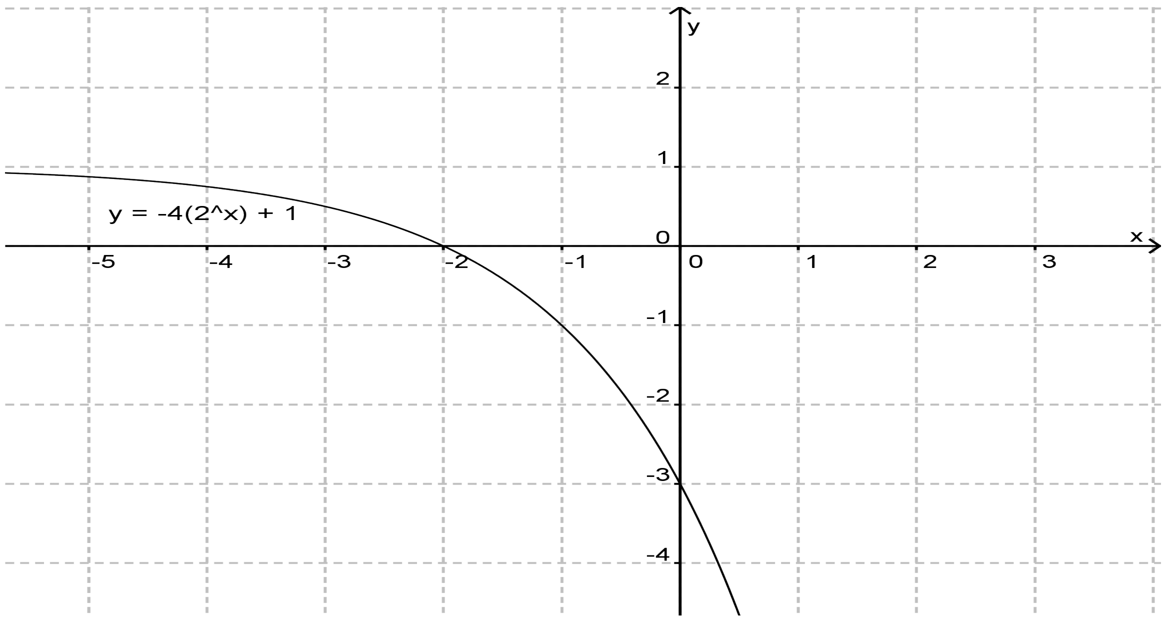
c)



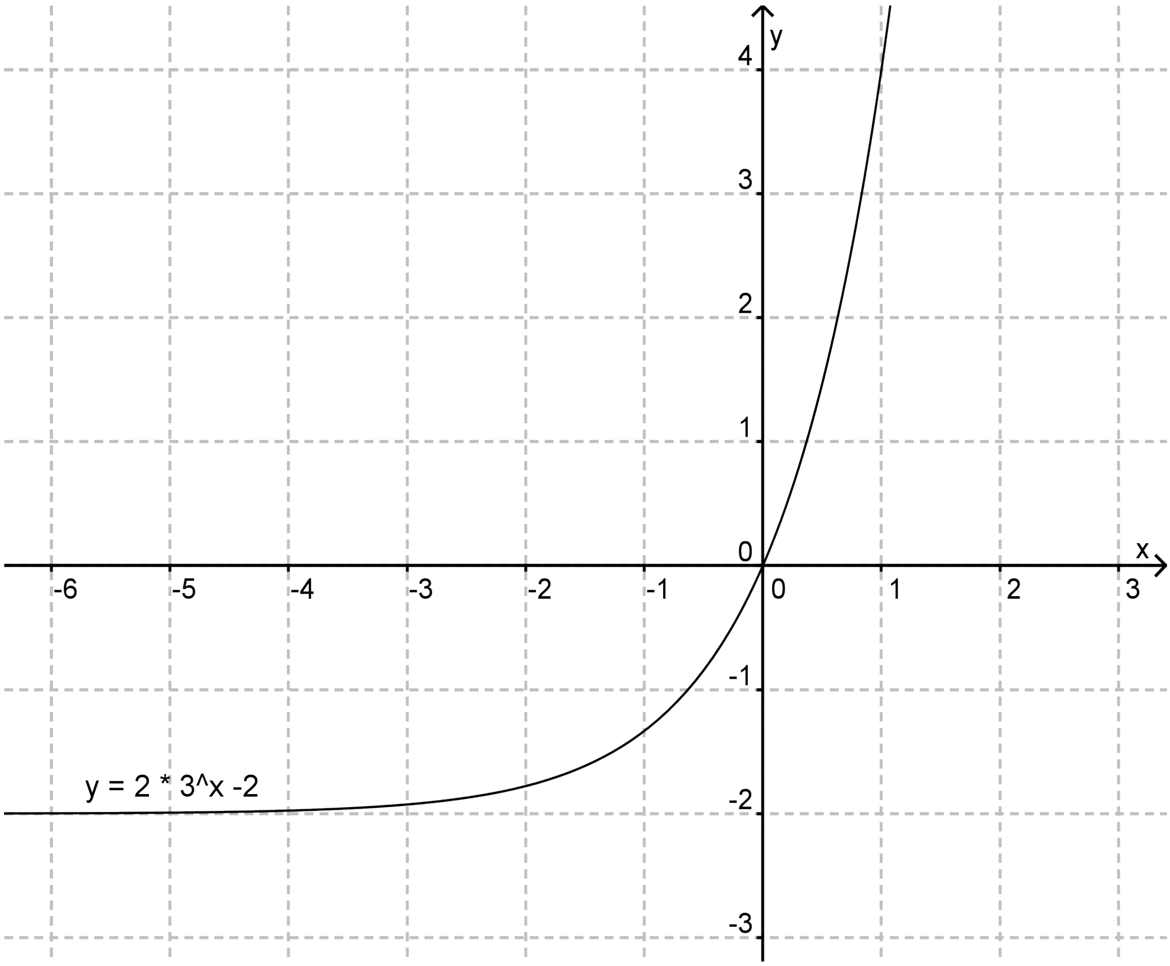
3a)


y
12
10
8
6
4
y = 3^x + 2
2
0
x
-10
-8
-6
-4
-2
0
2
4
6
8
-2

b)

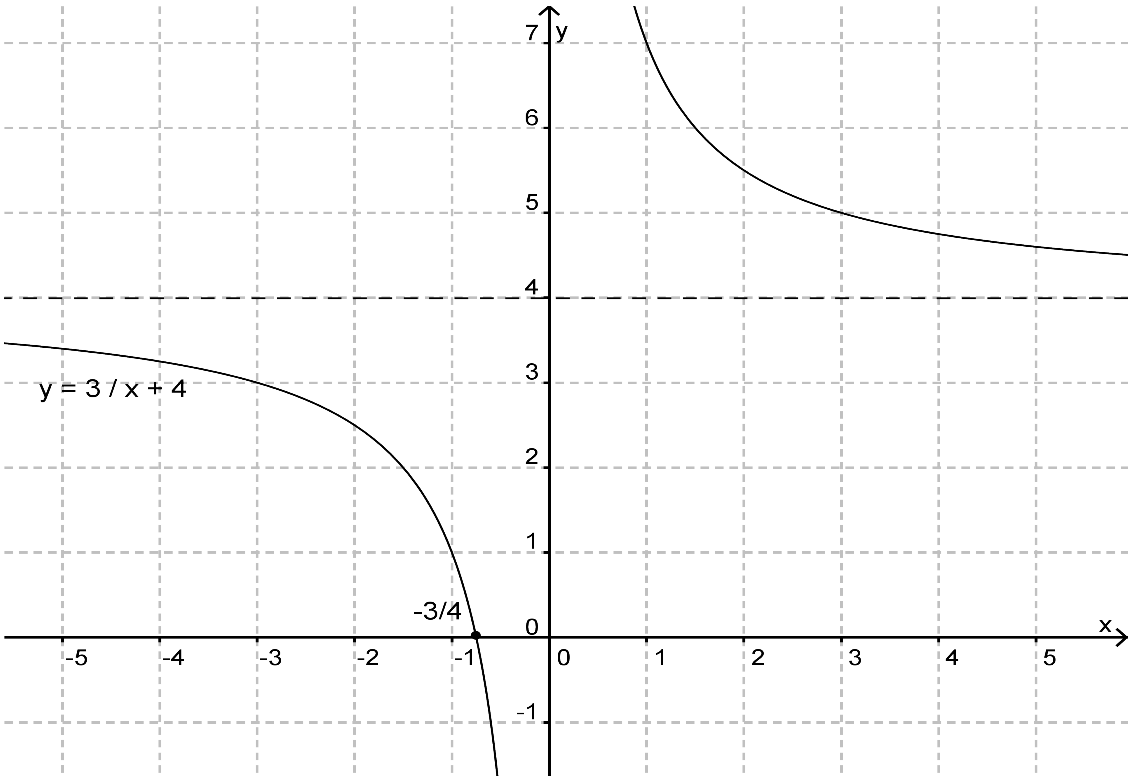

y
2
1
y = -4(2^x) + 1
0
x
-5
-4
-3
-2
-1
0
1
2
3
-1
-2
-3
-4

c)



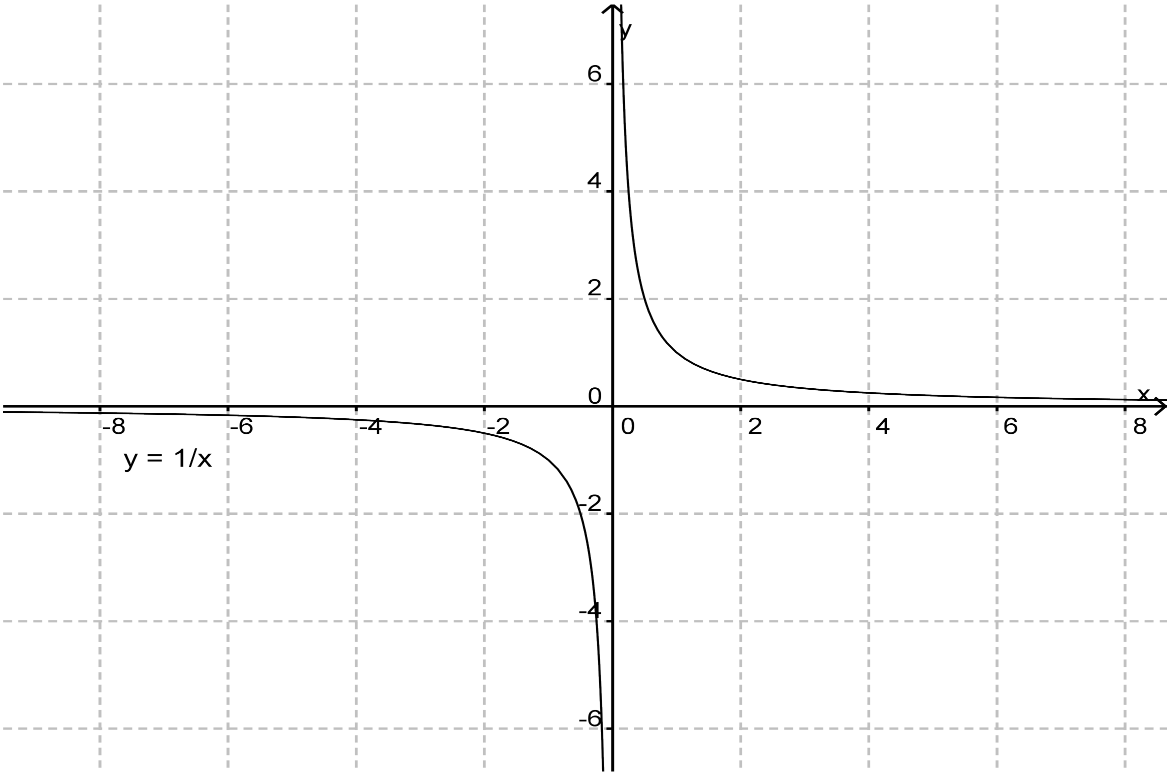
y
4
3
2
1
0
x
-6
-5
-4
-3
-2
-1
0
1
2
3
-1
y = 2 * 3^x -2
-2
-3

4. a)

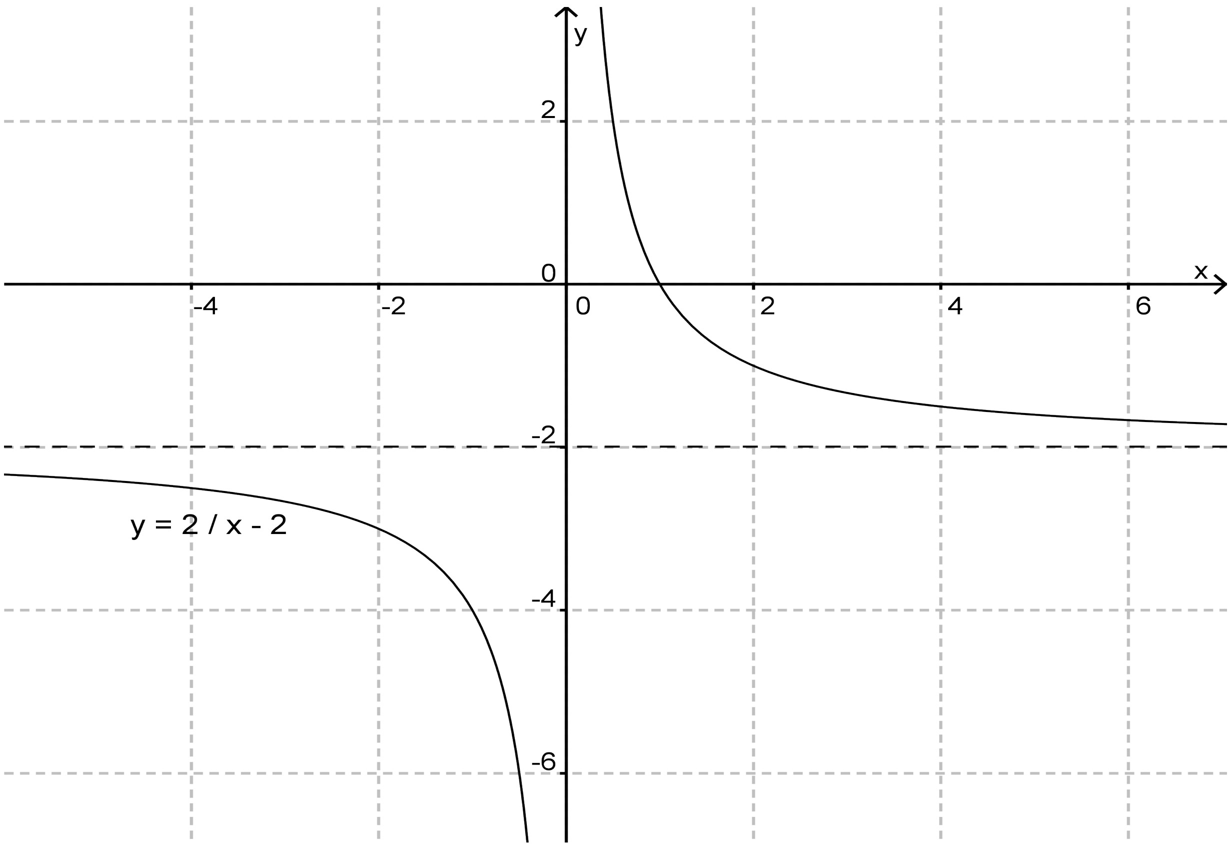


7
y
6
5
4
3
y = 3 / x + 4
2
1
-3/4
0
x
-5
-4
-3
-2
-1
0
1
2
3
4
5
-1

b)



c)



5. a) False – the given or chosen x-value is known as the independent variable.

b) False – an intercept is the point at which a graph intersects an axis.

c) True.

d) False – a graph is said to be continuous if there are no breaks in it.

e) True.

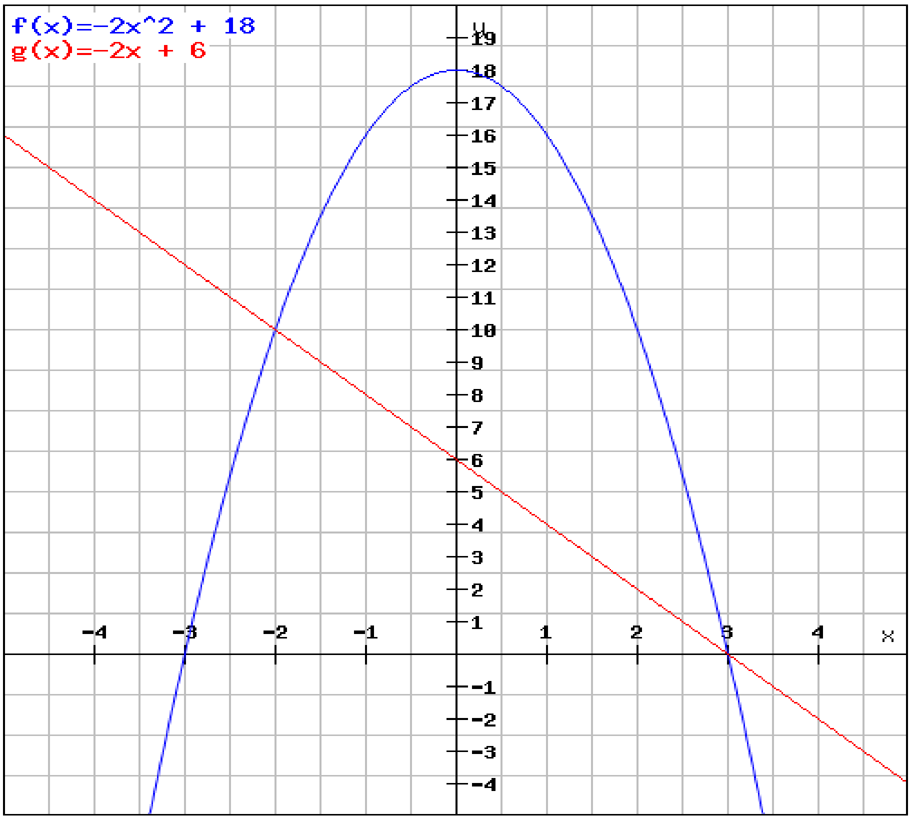
f) False – functions of the form $y=\frac{a}{x}+q$ are hyperbolic functions.

g) False – an asymptote is a straight or curved line which a graph will approach but never touch (or intersect!)

h) True.

i) False. The graph of a straight line never has a turning point. The graph of a parabola always has a turning point.

6.



b) The x-values of the points of intersection will be when f(x) = g(x).

$$-2x^2+18=-2x+6$$
$$x^2-9=x-3$$
$$x^2-x-6=0$$
$$(x+2)(x-3)=0$$

So x = -2 or x = 3.

The y-values can be obtained by substituting into either equation:

$g(x)=-2(3)+6=0$ or $g(x)=-2(-2)+6=10$

The points of intersection are: (3;0) and (-2;10)

$$f(x)>0$$
$$-2x^2+18>0$$
$$2x^2<18$$
$$x^2<9$$

ci) $-3<x<3$

ii)

d) $y=2x^2-18$

7. We substitute x = 5 into the equation:

$y=5(0,8)^5$

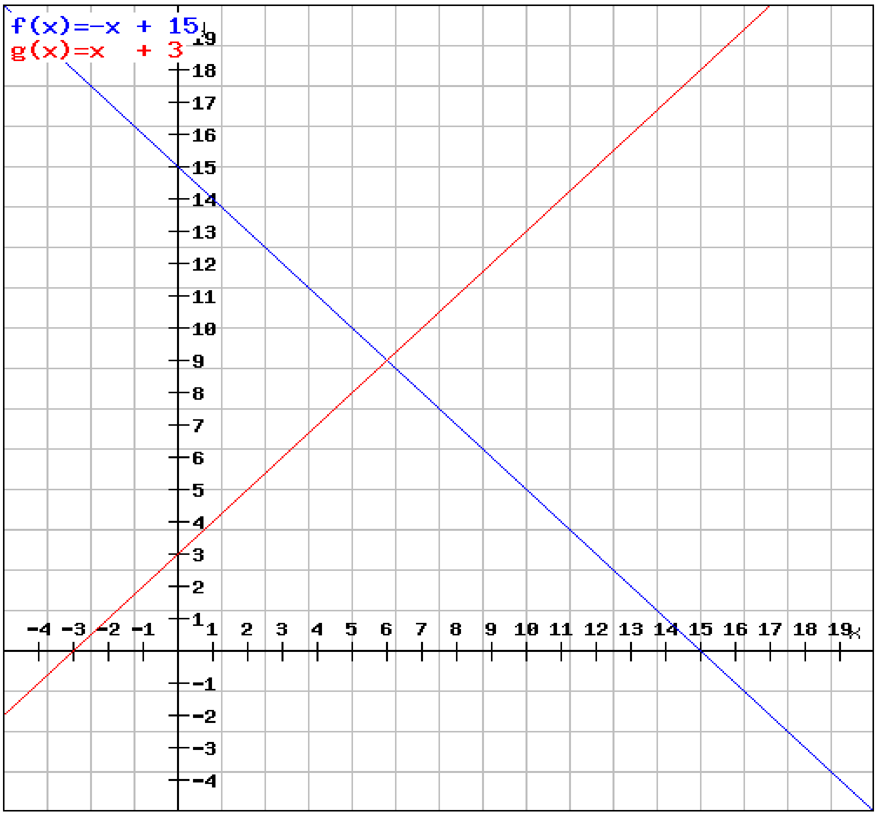
$y=1,6$

The approximate height of the fifth bounce is 1,6 units.

8. a) Let x = five rand coins and y = two rand coins.

$x + y = 15$ and $y = x + 3$ are the two equations.

c) c) To answer the question we look at the point of intersection of the graphs. From the graph we see that this occurs at (6;9)

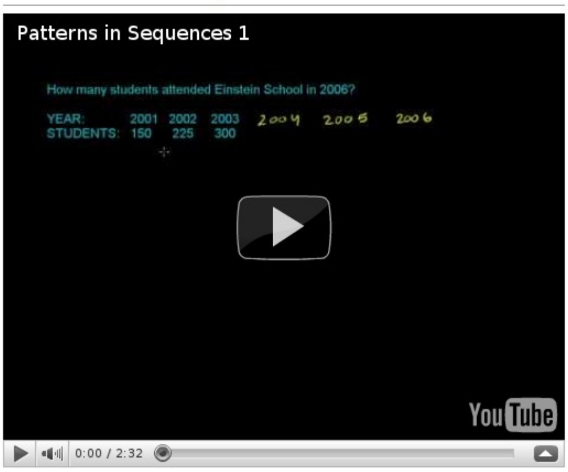


Chapter 3 - Number Patterns

http://cnx.org/content/m38376/latest/?collection=col11306/latest

Videos

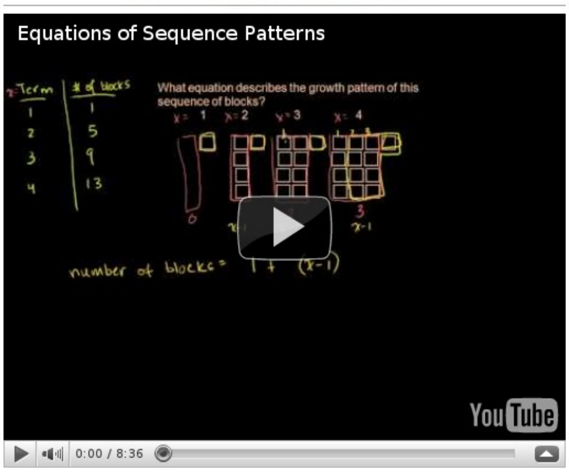
Khan Academy video on Number Patterns - 1



This Khan Academy video on Patterns in Sequences 1 covers an example of a number pattern and a simple way to work out the next few numbers in the pattern.

This video can be downloaded at: http://www.fhsst.org/laJ

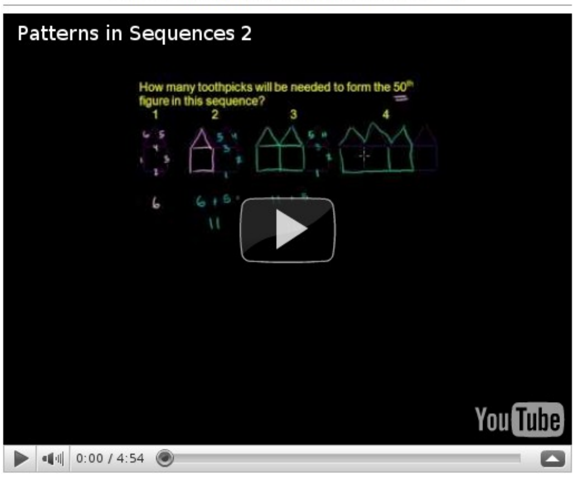
Khan Academy video on Number Patterns



This Khan Academy video on Equations of Sequence Patterns covers working out the equation for the sequence.

This video can be downloaded at: http://www.fhsst.org/laS

Khan Academy video on Number Patterns - 2



This Khan Academy video on Patterns in Sequences 2 covers an example of a sequence and working out a term of the sequence.

This video can be downloaded at: http://www.fhsst.org/lah

Solutions

Exercises

1. The first term is 3.
   The second term is: 3 + 4.
   The third term is: 3 + 4 + 4.
   The fourth term is: 3 + 4 + 4 + 4.
   Looking at this we see that each term adds (n – 1) 4's to the original term(3). Or: 4(n – 1) + 3.
   So the nth term must be: $a_n = 3 \quad 4 \quad n - 1$.

2. a) The first term is -2.

The second term is: -2 + 3.

The third term is: -2 + 3 + 3.

The fourth term is: -1 + 3 + 3 + 3.

Looking at this we see that each term adds (n – 1) 3's to the original term(-2). Or: 3(n – 1) - 2.

So the general formula must be: $a_n = -2 \quad 3 \quad n - 1$

b)The first term is 11.

The second term is:11 + 4.

The third term is: 11 + 4 + 4.

The fourth term is: 11 + 4 + 4 + 4.

Looking at this we see that each term adds (n – 1) 4's to the original term(11). Or: 4(n – 1) + 11.

So the general formula must be: $a_n = 11 \quad 4 \quad n - 1$

c) $a_n = a_1 + d(n-1)$

$$a_3 = 7 = a_1 + d(3-1) = a_1 + 2d$$

$$a_8 = 15 = a_1 + d(8-1) = a_1 + 7d$$

$$a_1 = 15 - 7d$$

$$7 = 15 - 7d + 2d$$

$$7 - 15 = -7d + 2d$$

$$-8 = -5d$$

$$d = \tfrac{8}{5}$$

$$7 = a_1 + 2\left(\tfrac{8}{5}\right)$$

$$a_1 = 7 - \tfrac{16}{5}$$

$$a_1 = \tfrac{19}{5}$$

So the general formula is:

$$a_n = \tfrac{19}{5} + \tfrac{8}{5}(n-1)$$

d) $a_n = a_1 + d(n-1)$

$$a_4 = -8 = a_1 + d(4-1) = a_1 + 3d$$

$$a_{10} = 10 = a_1 + d(10-1) = a_1 + 9d$$

$$a_1 = 10 - 9d$$

$$-8 = 10 - 9d + 3d$$

$$-8 - 10 = -9d + 3d$$

$$-18 = -6d$$

$$d = \tfrac{-18}{-6} = 3$$

$$-8 = a_1 + 3(3)$$

$$a_1 = -8 - 9$$

$$a_1 = -17$$

So the general formula is:

$$a_n = -17 + 3(n-1)$$

3. Extract the relevant information which is a sequence: 15, 19, 23...

Now find the general formula:

The first term is 15.

The second term is:15 + 4.

The third term is: 15 + 4 + 4.

Looking at this we see that each term adds (n – 1) 4's to the original term(15). Or: 4(n – 1) + 15.

So the general formula must be: $a_n = 15 \quad 4 \quad n - 1$.

Now we apply this with n = 25 to get the number of seats in row 25:

$$a_{25} = 15 \quad 4 \quad 25 - 1 \quad = 15 \quad 4 \quad 24 \quad = 15 \quad 96 = 111 \text{seats}$$

4. a) The first term is 4, since this is the number of matches needed for 1 square.

b) The common difference between the terms is 3. (7 – 4 = 3, 10 – 7 = 3)

c) $a_n = 4 \quad 3 \quad n - 1$

d) $a_{25} = 4 \quad 3 \quad 25 - 1 \quad = 76$

5. The general formula is $a_n = 5n$ where n is the number of weeks. So we want to find n.

$a_n = 50 = 5n$

$n = \frac{50}{5} = 10$. So after ten weeks you will deposit R50.

6.

With one line intersecting at four points we get five parts. If we add a second line it is now broken up into 9 parts. And if we add a third line it is now broken up into 13 parts. So we see that for each line added we add four parts. So the sequence is: 5; 9; 13; ….

Using:

$a_n=a_1 + d(n-1)$ with $a_1=5$ and d = 4 we get the following general formula:

$a_n=5 + 4(n-1)$

Substituting n=19 we get:

$a_{19}=5 + 4(19-1)=5 + 4(18)=5 + 72=77$

So the string will be divided into 19 parts.

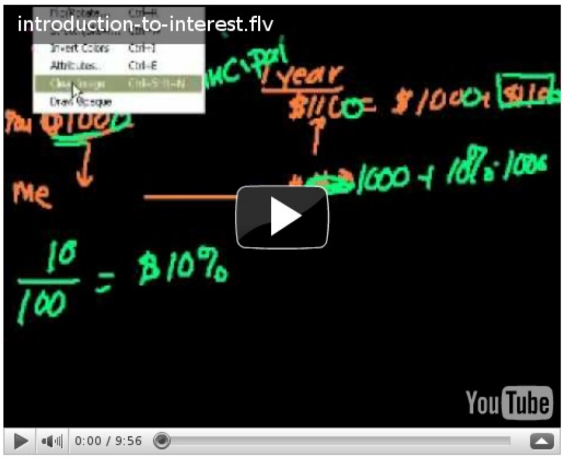
Chapter 4 - Finance

http://cnx.org/content/m38360/latest/?collection=col11306/latest

Videos

Note that in the following videos although the examples are done using dollars, we can use the fact that dollars are a decimal currency and so are interchangeable (ignoring the exchange rate) with rands. This is what is done in the subtitles.

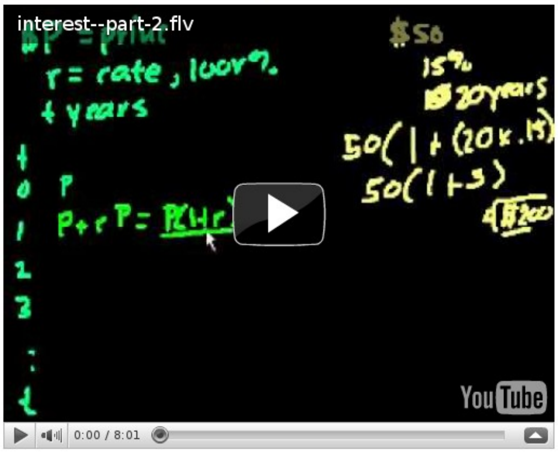
Khan academy video on interest - 1



This Khan Academy video on Introduction to Interest provides an overview of the concept of interest and explains how compound and simple interest differ. This video does not go into details of how to calculate interest from equations, but rather just gives an intuitive feel for interest.

This video can be downloaded at: http://www.fhsst.org/lbi

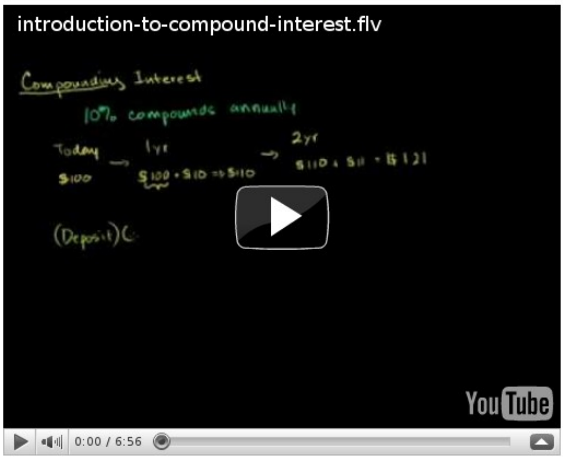
Khan academy video on interest - 2



This Khan Academy video on Interest part 2 covers a more general approach to interest and how to work out interest for any given starting values.

This video can be downloaded at: http://www.fhsst.org/lb3

Khan academy video on interest - 3



This Khan Academy video on Introduction to Compound Interest covers the concept of compound interest in more depth. An explanation of how to work out compound interest is explored.

This video can be downloaded at: http://www.fhsst.org/lbO

Solutions

Simple Interest

1. $$\begin{aligned} A &= P(1+i\cdot n)\\ A &= 3500(1+(2)(0{,}075))\\ A &= R4025 \end{aligned}$$

2.

a) $$\begin{aligned} \text{interest} &= P \quad 1 \quad i\cdot n \quad - P\\ &= 300 \quad 1 \quad 0{,}08\times 1 \quad - 300\\ &= 324 - 300 = 24 \end{aligned}$$

b) $$\begin{aligned} \text{interest} &= P \quad 1 \quad i\cdot n \quad - P\\ &= 225 \quad 1 \quad 0{,}125\times 6 \quad - 225\\ &= 393{,}75 - 225 = 168{,}75 \end{aligned}$$

3. $$\begin{aligned} A &= P \quad 1 \quad i\cdot n\\ 18000 &= 5000 \quad 1 \quad 0{,}16i\\ \tfrac{18000}{5000} &= 1 \quad 0{,}16i\\ 3{,}6 &= 1 \quad 0{,}16i\\ 2{,}6 &= 0{,}16i\\ i &= 16{,}25\,\% \end{aligned}$$

4.

a) $$\begin{aligned} A &= P \quad 1 \quad i\cdot n\\ &= 8500 \quad 1 \quad 0{,}175 \quad n\\ &= R\ 12962{,}50 \end{aligned}$$

b) $R\ 12962{,}50 - R\ 8500 = R\ 4462{,}50$

c) Total of 36 months:

$\frac{12962{,}50}{36} = R\ 360{,}07$

Compound Interest

1. $$A = P\ \ 1\ \ \ i\ \ ^{n}$$
$$A = 3500\ \ 1\ \ \ 0{,}075\ \ ^{2}$$
$$A = \text{R}\,4044{,}69$$

2. $$A = P\ \ 1\ \ \ i\ \ ^{n}$$
$$A = 1425\ \ 1\ \ \ 0{,}073\ \ ^{6}$$
$$A = \text{R}\,2174{,}77$$

3. $$A = P\ \ 1\ \ \ i\ \ ^{n}$$
$$100000 = P\ \ 1\ \ \ 0{,}11\ \ ^{5}$$
$$P = \text{R}\,59345{,}13$$

Foreign Exchange

1. a) Cost in rands = (cost in pounds) times exchange rate
$\text{Cost i rands} = 100 \times \frac{14}{1} = \text{R}1400$
b) $\text{Cost i rands} = 100 \times \frac{12}{1} = \text{R}1200$. So you will save R200 (Saving = R1400 - R1200).
c) $\text{Cost i rands} = 100 \times \frac{15}{1} = \text{R}1500$. So you will lose R100. (Loss = R1400 - R1500)

2. a) To answer this question we work out the cost of the car in rand for each country and then compare the three answers to see which is the cheapest. Cost in rands = cost in currency times exchange rate.
Cost in UK: $12200 \times \frac{14{,}13}{1} = \text{R}172386$
Cost in USA: $21900 \times \frac{7{,}04}{1} = \text{R}154400$
Comparing the three costs we find that the car is the cheapest in the USA.
b) Sollie: $6 \times \frac{14{,}13}{1} = \text{R}84{,}78$
Arinda: $12 \times \frac{7{,}04}{1} = \text{R}84{,}48$

End Of Chapter Exercises

1. $$\text{cost in rands} = \text{cost in euros} \times \text{R} / \text{euro exchange rate}$$
$$= 200 \times \frac{9{,}20}{1}$$
$$= \text{R}\,1840$$

2. a) $$A = P\ \ 1\ \ \ i \cdot n$$
$$A = 500\ \ 1\ \ \ \ 0{,}0685\ \ \ \ 1$$
$$A = \text{R}\,534{,}25$$
b) $$A = P\ \ 1\ \ \ i\ \ ^{n}$$
$$A = 500\ \ 1\ \ \ 0{,}04\ \ ^{1}$$
$$A = \text{R}\,520$$

3.

$$A = P \quad 1 \quad i \cdot n$$
$$A = 1450 \quad 1 \qquad 0{,}11 \qquad 3$$

Bank A: $A = \text{R } 1925{,}50$

$$A = P \;\; 1 \;\; i \;^{n}$$
$$A = 1450 \;\; 1 \;\; 0{,}105 \;^{3}$$

Bank B: $A = \text{R } 1956{,}39$

She should choose Bank B as it will give her more money after 3 years.

4.

$$\text{interest} = P \quad 1 \quad i \cdot n \quad - P$$
$$= 2000 \quad 1 \qquad 1 \qquad 0{,}11 \qquad - 2000$$
$$= 200$$

5.

$$\text{interest} = P \;\; 1 - i \;^{n} - P$$
$$= 2000 \;\; 1 - 0{,}11 \;^{1} - 2000$$
$$= 200$$

6.

a) Simple interest. Interest is only calculated on the principal amount and not on the interest earned during prior periods. This will lead to the borrower paying less interest.

b) Compound interest. Interest is calculated from the principal amount as well as interest earned from prior periods. This will lead to the banker getting more money for the bank.

7.

a)
$$\text{interest} = P \;\; 1 \;\; i \;^{n} - P$$
$$= 1500 \;\; 1 \;\; 0{,}05 \;^{2} - 1500$$
$$= 2205 - 1500$$
$$= 205$$

b)
$$\text{interest} = P \;\; 1 \;\; i \;^{n} - P$$
$$= 1500 \;\; 1 \;\; 0{,}06 \;^{3} - 1500$$
$$= 1786{,}53 - 1500$$
$$= 286{,}53$$

c)
$$\text{interest} = P \;\; 1 \;\; i \;^{n} - P$$
$$= 800 \;\; 1 \;\; 0{,}16 \;^{1} - 800$$
$$= 925 - 800$$
$$= 125$$

8.

$$\text{AUD / Yen} = \text{ZAR / Yen} \times \text{AUD / Yen}$$
$$= \frac{6{,}2287}{100} \times \frac{1}{5{,}1094}$$
$$= 0{,}01219$$
$$0{,}00219 \text{ AUD} = 1 \text{ Yen}$$
$$\text{or } 1 \text{ AUD} = 82{,}03 \text{ Yen}$$

9.

$$\text{Total paid } = 3750 \quad 956{,}25 = 4706{,}25$$

$$A = P \quad 1 \quad i \cdot n$$

$$4706{,}25 = 3750 \quad 1 \quad 3i$$

$$1 \quad 3i = \frac{4706{,}25}{3750}$$

$$3i = 0{,}255$$

$$i = 0{,}085$$

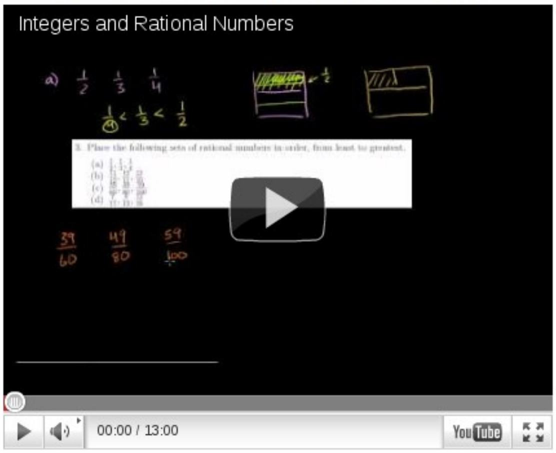
So i = 8,5%

Chapter 5 - Rational Numbers

http://cnx.org/content/m38348/latest/?collection=col11306/latest

Videos

Khan Academy video on Integers and Rational Numbers



This Khan Academy video on Integers and Rational Numbers explains some of the basic concepts of integers and rational numbers. Salman Khan works through some problems on integers and rational numbers.

This video can be downloaded at: http://www.fhsst.org/laL

Solutions

Rational Numbers

1. (a) rational
   (b) rational
   (c) rational
   (d) irrational
2. (a) valid
   (b) valid
   (c) invalid
   (d) valid, because $\frac{2,1}{1} = \frac{21}{10}$

Fractions

1. a) $\frac{1}{10}$
   (b) $0,12 = \frac{12}{100} = \frac{3}{25}$
   (c) $0,58 = \frac{58}{100} = \frac{29}{50}$
   (d) $0,2589 = \frac{2589}{10000}$

Repeated Decimal Notation

1. (a) $0,11111111\ldots\ = 0,\dot{1}$
   (b) $0,1212121212\ldots\ = 0,\overline{12}$
   (c) $0,123123123123\ldots\ = 0,\overline{123}$
   (d) $0,11414541454145\ldots\ = 0,114\overline{145}$

2. (a) $\frac{2}{3} = 2(\frac{1}{3}) = 2(0,3333333\ldots) = 0,666666\ldots\ = 0,\dot{6}$
   (b) $1\frac{3}{11} = 1 + 3(\frac{1}{11}) = 1 + 3(0,090909\ldots) = 1 + 0,27272727\ldots\ = 1,\overline{27}$
   (c) $4\frac{5}{6} = 4 + 5(\frac{1}{6}) = 4 + 5(0,166666\ldots) = 4 + 0,833333\ldots\ = 4,8\dot{3}$
   (d) $2\frac{1}{9} = 2 + 0,1111111\ldots\ = 2,\dot{1}$

3. (a)

$$10x = 6,333\dot{3} \ \ldots 1$$
$$100x = 63,333\dot{3} \ \ldots 2$$

subtracting 1 from 2 gives:

$$90x = 57$$
$$x = \frac{57}{90}$$

(b)

$$x = 5,3131\overline{31} \ \ldots 1$$
$$100x = 531,3131\overline{31} \ \ldots 2$$

subtracting 1 from 2 gives:

$$99x = 526$$
$$x = \frac{526}{99}$$

(c)

$$x = 0,99999\dot{9} \ \ldots 1$$
$$10x = 9,99999\dot{9} \ \ldots 2$$

subtracting 1 from 2 gives:

$$9x = 9$$
$$x = \frac{9}{9}$$
$$= 1$$

so $0,99999\dot{9} = 1$

End of Chapter Exercises

1. $\frac{5}{2}$, $\frac{a}{3}$, $\frac{b}{2}$ are all rational. $\frac{1}{c}$ is not rational since c is an irrational number.

2. a) $\frac{5}{10}$ or $\frac{1}{2}$
   b) $\frac{1}{10} + \frac{2}{100} = \frac{12}{100} = \frac{6}{10} = \frac{3}{5}$
   c) $\frac{5}{10} + \frac{8}{100} = \frac{58}{100} = \frac{29}{50}$
   d) $\frac{6}{10} = \frac{3}{5}$
   e) $1 + \frac{5}{10} + \frac{9}{100} = 1\frac{59}{100}$

3.

$$x = 3.21\dot{1}\dot{8}$$
$$x = 3.2118181818\ldots$$
$$10000x = 32118.181818\ldots$$
$$9999x = 32115$$
$$x = \frac{32115}{9999}$$

This is a rational number since both the numerator and the denominator are integers.

4.

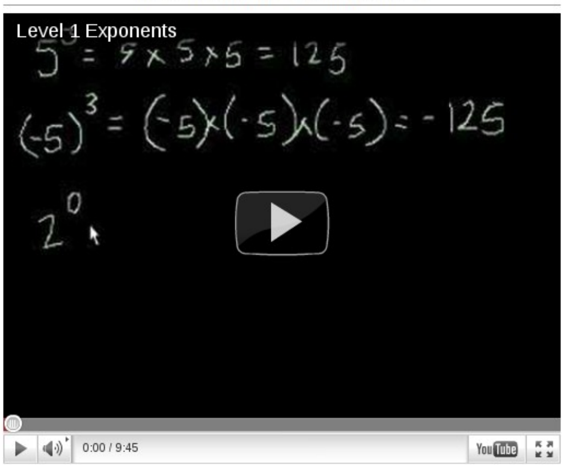
$$x = 0,7878787878\ldots.$$
$$100x = 78,78787878\ldots$$
$$99x = 78$$
$$x = \frac{78}{99}$$

Chapter 6 - Exponentials

http://cnx.org/content/m38359/latest/?collection=col11306/latest

Videos

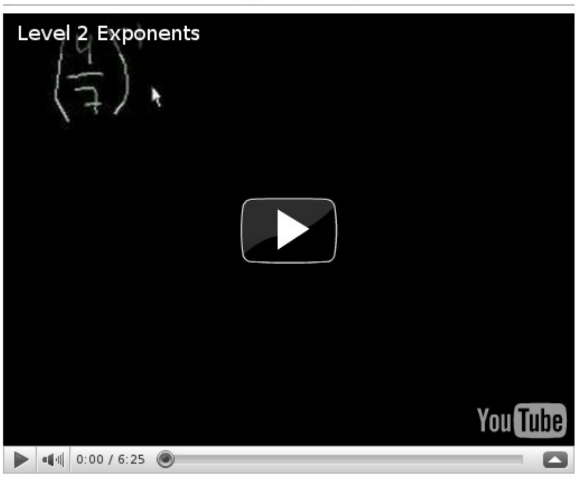
Khan Academy video on Exponents - 1



This Khan Academy video on Exponents Level 1 gives an introduction to the concept of exponents and shows a few simple examples on exponents and how to calculate them.

This video can be downloaded at: http://www.fhsst.org/laM

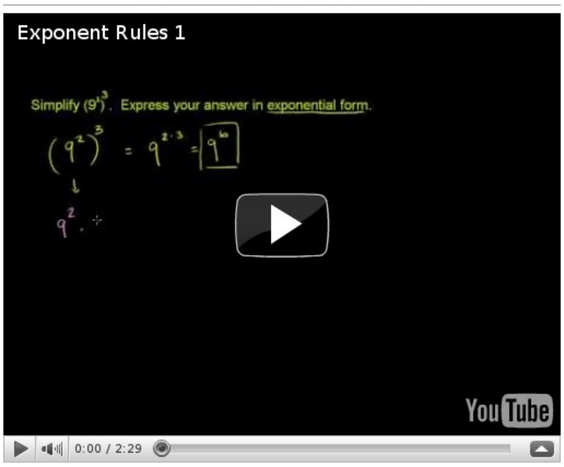
Khan Academy video on Exponents-2



This Khan Academy video on Exponents Level 2 takes the basic concept of exponents further by introducing negative exponents.

This video can be downloaded at: http://www.fhsst.org/lae

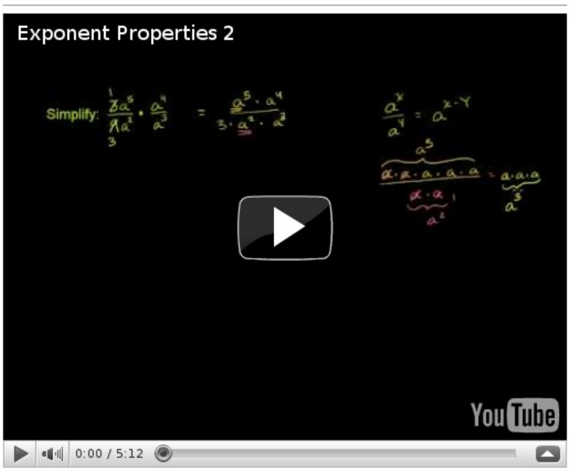
Khan Academy video on Exponents - 3



This Khan Academy video on Exponents Law 1 covers some of the laws of exponents. Salman Khan works through a problem on exponents and shows how to apply the exponent laws to the problem.

This video can be downloaded at: http://www.fhsst.org/lat

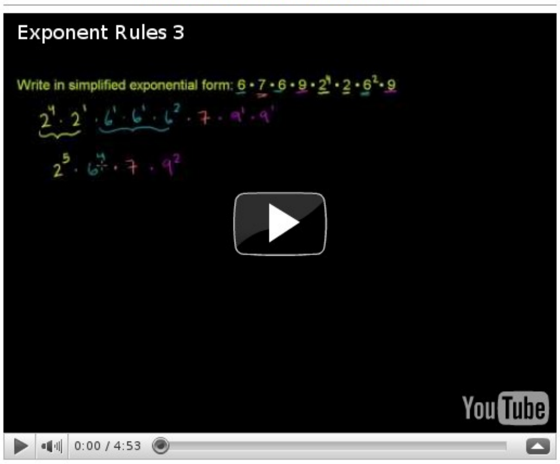
Khan academy video on exponents - 4



This Khan Academy video on Exponent Properties 2 covers some more of the laws of exponents. This video explains how to deal with exponents in the denominator of a fraction.

This video can be downloaded at: http://www.fhsst.org/laz

Khan Academy video on Exponents - 5



This Khan Academy video on Exponent Rules 3 covers several of the laws of exponents. This video works through a problem involving mixed bases and how to simplify expressions with mixed bases.

This video can be downloaded at: http://www.fhsst.org/lau

Solutions

Applications using Law 1

1. $16^0=1$
2. $16a^0=16(1)=16$
3. $(16+a)^0=1$
4. $(-16)^0=1$
5. $-16^0=-1\times16^0=-1$

Application using Law 2

1. $x^2.x^5=x^{2+5}=x^7$
2. $2x^3y\times5x^2y^7=2(5)x^{3+2}y^{1+7}=10x^5y^8$
3. $2^3.2^4=2^{3+4}=2^7$
4. $3\times3^{2a}\times3^2=3^{1+2a+2}=3^{2a+3}$

Application using Law 3

1. $2^{-2}=\frac{1}{2^2}=\frac{1}{4}$
2. $\frac{2^{-2}}{3^2}=\frac{1}{2^2.3^2}=\frac{1}{4\times9}=\frac{1}{36}$
3. $\left(\frac{2}{3}\right)^{-3}=\frac{2^{-3}}{3^{-3}}=\frac{3^3}{2^3}=\frac{27}{8}$
4. $\frac{m}{n^{-4}}=mn^4$
5. $\frac{a^{-3}.x^4}{a^5.x^{-2}}=\frac{x^2.x^4}{a^5.a^3}=\frac{x^{2+4}}{a^{5+3}}=\frac{x^6}{a^8}$

Application using Law 4

1. $\frac{a^6}{a^2}=a^{6-2}=a^4$
2. $\frac{3^2}{3^6}=3^{2-6}=3^{-4}=\frac{1}{3^4}$(always give final answer with the positive index)
3. $\frac{32a^2}{4a^8}=8a^{2-8}=8a^{-6}=\frac{8}{a^6}$
4. $\frac{a^{3x}}{a^4}=a^{3x-4}$

Application using Law 5

1. $(2x^2y)^3=2^3x^{2\times3}y^5=8x^6y^5$
2. $\left(\frac{7a}{b^3}\right)^2=\frac{7^2a^2}{b^3}=\frac{49a^2}{b^3}$
3. $(5a)^3=5^3.a^3=125a^3$

Application using Law 6

1. $(x^3)^4=x^{3\times4}=x^{12}$
2. $[(a^4)^3]^2=a^{4\times3\times2}=a^{24}$
3. $(3^{n+3})^2=3^{2(n+3)}=3^{2n+6}$

End Of Chapter Exercises

1. (a) $302^0=1$
   (b) $1^0=1$
   (c) $(xyz)^0=1$
   (d) $[(3x^4y^7z^{12})^5(-5x^9y^3z^4)^2]^0=1$
   (e) $(2x)^3=2^3x^3=8x^3$
   (f) $(-2x)^3=(-2)^3x^3=-8x^3$
   (g) $(2x)^4=2^4x^4=16x^4$
   (h) $(-2x)^4=(-2)^4x^4=16x^4$

2.

(a) $\frac{3x^{-3}}{(3x)^2}=\frac{3}{3^2x^2x^3}=\frac{1}{3x^{2+3}}=\frac{1}{3x^5}$

(b)
$$
\begin{aligned}
& 5x^0+8^{-2}-\left(\tfrac{1}{2}\right)^{-2}.1^x \\
= \quad & 5.1+\tfrac{1}{8^2}-\tfrac{1^{-2}}{2^{-2}}.1 \\
= \quad & 5+\tfrac{1}{64}-2^2 \\
= \quad & 5+\tfrac{1}{64}-4 \\
= \quad & \tfrac{65}{64}
\end{aligned}
$$

(c) $\frac{5^{b-3}}{5^{b+1}}=5^{b-3-b-1}=5^{-4}=\frac{1}{5^4}=\frac{1}{625}$

3.

(a) $\frac{2^{a-2}.3^{a+3}}{6^a}=\frac{2^{a-2}.3^{a+3}}{(2.3)^a}=\frac{2^{a-2}.3^{a+3}}{2^a.3^a}=2^{a-2-a}.3^{a+3-a}=2^{-2}.3^3=\frac{3^3}{2^2}$

(b) $\frac{a^{2m+n+p}}{a^{m+n+p}.a^m}=a^{2m+n+p-(m+n+p)-m}=a^{2m+n+p-m-n-p-m}=a^0=1$

(c) $\frac{3^n.9^{n-3}}{27^{n-1}}=\frac{3^n.(3^2)^{n-3}}{(3^3)^{n-1}}=\frac{3^n.3^{2n-6}}{3^{3n-3}}=3^{n+2n-6-(3n-3)}=3^{3n-6-3n+3}=3^{-3}=\frac{1}{3^3}$

(d) $\left(\frac{2x^{2a}}{y^{-b}}\right)^3=\frac{2^3(x^{2a})^3}{(y^{-b})^3}=\frac{2^3x^{6a}}{y^{-3b}}=2^3x^{6a}y^{3b}$

(e) $\frac{2^{3x-1}.8^{x+1}}{4^{2x-2}}=\frac{2^{3x-1}.(2^3)^{x+1}}{(2^2)^{2x-2}}=\frac{2^{3x-1}.2^{3x+3}}{2^{4x-4}}=2^{3x-1+3x+3-4x+4}=2^{2x+6}$

(f)
$$
\begin{aligned}
\frac{6^{2x}.11^{2x}}{22^{2x-1}.3^{2x}} \quad = \quad & \frac{(2.3)^{2x}.11^{2x}}{(2.11)^{2x-1}.3^{2x}} \\
= \quad & \frac{2^{2x}.3^{2x}.11^{2x}}{2^{2x-1}.11^{2x-1}.3^{2x}} \\
= \quad & 2^{2x-2x+1}.3^{2x-2x}.11^{2x-2x+1} \\
= \quad & 2^1.3^0.11^1 \\
= \quad & 2.1.11 \\
= \quad & 22
\end{aligned}
$$

4.

(a) $\frac{(-3)^{-3}.(-3)^2}{(-3)^{-4}}=(-3)^{-3+2+4}=(-3)^3=(-1)^3.3^3=-1.27=-27$

(b) $(3^{-1}+2^{-1})^{-1}=\left(\frac{1}{3}+\frac{1}{2}\right)^{-1}=\left(\frac{2}{6}+\frac{3}{6}\right)^{-1}=\left(\frac{5}{6}\right)^{-1}=\left(\frac{5^{-1}}{6^{-1}}\right)=\frac{6}{5}$

(c)
$$
\begin{aligned}
\frac{9^{n-1}.27^{3-2n}}{81^{2-n}} \quad = \quad & \frac{(3^2)^{n-1}.(3^3)^{3-2n}}{(3^4)^{2-n}} \\
= \quad & \frac{3^{2n-2}.3^{9-6n}}{3^{8-4n}} \\
= \quad & 3^{2n-2+9-6n-(8-4n)} \\
= \quad & 3^{2n-2+9-6n-8+4n} \\
= \quad & 3^{2n-6n+4n-2+9-8} \\
= \quad & 3^{-1} \\
= \quad & \frac{1}{3}
\end{aligned}
$$

(d)
$$
\begin{aligned}
\frac{2^{3n+2}.8^{n-3}}{4^{3n-2}} \quad = \quad & \frac{2^{3n+2}.(2^3)^{n-3}}{(2^2)^{3n-2}} \\
= \quad & \frac{2^{3n+2}.2^{3n-9}}{2^{6n-4}} \\
= \quad & 2^{3n+2+3n-9-(6n-4)} \\
= \quad & 2^{3n+2+3n-9-6n+4} \\
= \quad & 2^{-3} \\
= \quad & \frac{1}{2^3}
\end{aligned}
$$

Chapter 7 - Estimating Surds

http://cnx.org/content/m38347/latest/?collection=col11306/latest

Solutions

End of Chapter Exercises

1.
    a. 2 and 3
    b. 3 and 4
    c. 4 and 5
    d. 5 and 6
    e. 1 and 2
    f. 2 and 3
    g. 3 and 4
    h. 3 and 4
2. 2; 3
3. 3; 4

Chapter 8 - Irrational Numbers and Rounding Off

http://cnx.org/content/m38349/latest/?collection=col11306/latest

Solutions

End of Chapter Exercises

1.
a) To write to two decimal places we must convert to decimal. $\frac{1}{2} = 0{,}50$
b) To write to two decimal places just add a comma and two 0's: 1,00
c) We mark where the cut off point is, determine if it has to be rounded up or not and then write the answer. In this case there is a 1 after the cut off point so we do not round up. The final answer is: $0{,}111111 \approx 0{,}11$
d) Repeat the steps in c) but this time we round up. The answer is: $0{,}999991 \approx 1{,}00$

2. We mark where the cut off point is, determine if it has to be rounded up or not and then write the answer.
a) 3,14 (no rounding as there is a 1 after the cut off point)

b) 1,62 (round up as there is a 8 after the cut off point)

c) 1,41 (no rounding as there is a 4 after the cut off point)

d) 2,72 (round up as there is a 2 after the cut off point)

3. a) $1{,}414$
b) $1{,}732$
c) $2{,}236$
d) $2{,}449$

4. a) 2,82843 irrational number
b) 27,71281 irrational number

c) 10,00000 rational number

d) 0,70000 rational number

e) 0,04000 rational number

f) 0,500000 rational number

g) 6,00000 rational number

h) 44,27189 irrational number

i) 0,06000 rational number

j) -8(0,2) = -4,00000 rational number

k) 44,72136 irrational number

5. a) $3{,}142 = 3\frac{142}{1000} = 3\frac{71}{500}$

b) $1{,}618 = 1\frac{618}{1000} = 1\frac{309}{500}$

c) $1{,}414 = 1\frac{414}{1000} = 1\frac{207}{500}$

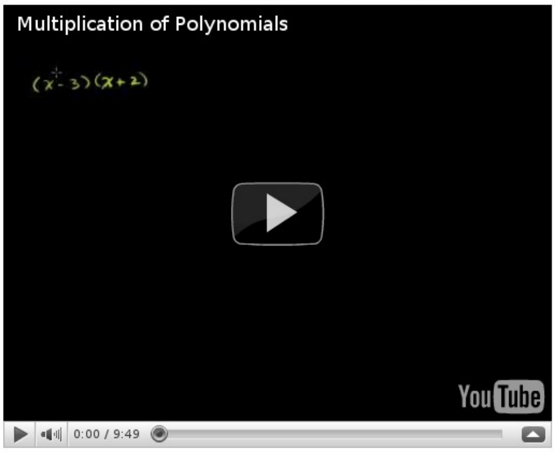
d) $2{,}718 = 2\frac{718}{1000} = \frac{359}{500}$

Chapter 9 - Products and Factors

http://cnx.org/content/m38375/latest/?collection=col11306/latest

Videos

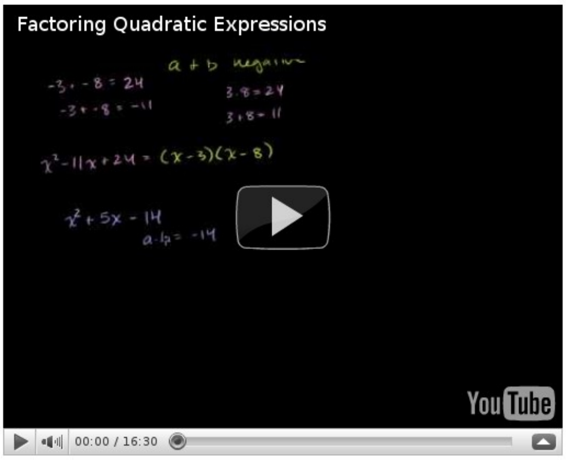
Khan Academy video on products of polynomials.



This Khan Academy video on Multiplication of Polynomials covers the basic concepts of how to multiply polynomials. This video works through several examples of multiplying polynomials increasing in difficulty.

This video can be downloaded at: http://www.fhsst.org/IC3

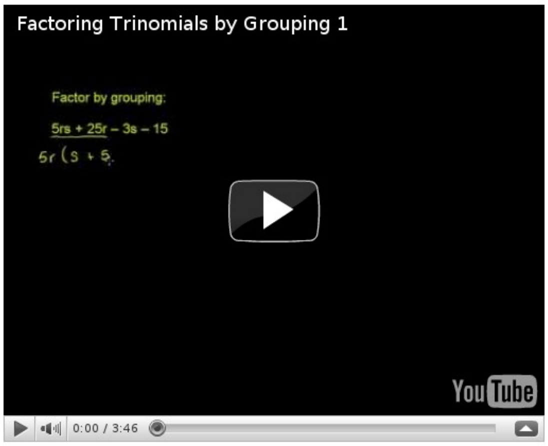
Khan Academy video on factorising a quadratic.



This Khan Academy video on Factoring Quadratic Expressions covers factoring quadratics. This video works through several examples of factoring quadratics and how to factor out quadratic expressions.

This video can be downloaded at: http://www.fhsst.org/ICO

Khan Academy video on factorising a trinomial by grouping.



This Khan Academy video on Factoring Trinomials by Grouping covers an example of factoring trinomials. This short video explains how to work out the factors of a trinomial and how to group the factors.

This video can be downloaded at: http://www.fhsst.org/lCc

Solutions

Recap

1.
$$\begin{aligned} 2y(y+4) &= 2y(y)+2y(4) \\ &= 2y^2+8y \end{aligned}$$

(a)
$$\begin{aligned} (y+5)(y+2) &= y(y+2)+5(y+2) \\ &= y(y)+y(2)+5(y)+5(2) \\ &= y^2+2y+5y+10 \\ &= y^2+7y+10 \end{aligned}$$

(b)
$$\begin{aligned} (y+2)(2y+1) &= y(2y+1)+2(2y+1) \\ &= y(2y)+y(1)+2(2y)+2(1) \\ &= 2y^2+y+4y+2 \\ &= 2y^2+5y+2 \end{aligned}$$

(c)
$$\begin{aligned} (y+8)(y+4) &= y(y+4)+8(y+4) \\ &= y(y)+y(4)+8(y)+8(4) \\ &= y^2+4y+8y+32 \\ &= y^2+12y+32 \end{aligned}$$

(d)
$$\begin{aligned} (2y+9)(3y+1) &= 2y(3y+1)+9(3y+1) \\ &= 2y(3y)+2y(1)+9(3y)+9(1) \\ &= 6y^2+2y+27y+9 \\ &= 6y^2+29y+9 \end{aligned}$$

(e)
$$\begin{aligned} (3y-2)(y+6) &= 3y(y+6)-2(y+6) \\ &= 3y(y)+3y(6)-2(y)-2(6) \\ &= 3y^2+18y-2y-12 \\ &= 3y^2+16y-12 \end{aligned}$$

(f)

2. (a) $$2l + 2w = 2(l + w)$$

(b) $$\begin{aligned} 12x + 32y &= 4(3)(x) + 4(8)(y) \\ &= 4(3x + 8y) \end{aligned}$$

(c) $$\begin{aligned} 6x^2 + 2x + 10x^3 &= 2x(3x) + 2x(1) + 2x(5x^2) \\ &= 2x(3x + 1 + 5x^2) \end{aligned}$$

(d) $$\begin{aligned} 2xy^2 + xy^2z + 3xy &= xy(2y) + xy(yz) + xy(3) \\ &= xy(2y + yz + 3) \\ &= xy[y(2 + z) + 3] \end{aligned}$$

(e) $$\begin{aligned} -2ab^2 - 4a^2b &= -2ab(b) - 2ab(2a) \\ &= -2ab(b + 2a) \end{aligned}$$

3. (a) $$7a + 4$$

(b) $$20a - 10 = 10(10a - 1)$$

(c) $$18ab - 3bc = 3b(6a - c)$$

(d) $$\begin{aligned} 12kj + 18kq &= 6k(2j) + 6k(3q) \\ &= 6k(2j + 3q) \end{aligned}$$

(e) $$\begin{aligned} 16k^2 - 4k &= 4k(4k) + 4k(-1) \\ &= 4k(4k - 1) \end{aligned}$$

(f) $$\begin{aligned} 2y(y + 4) &= 2y(y) + 2y(4) \\ &= 2y^2 + 8y \end{aligned}$$

(g) $$\begin{aligned} -6a - 24 &= -6(a) - 6(4) \\ &= -6(a + 4) \end{aligned}$$

(h) $$\begin{aligned} -2ab - 8a &= -2a(b) - 2a(4) \\ &= -2a(b + 4) \end{aligned}$$

(i) $$\begin{aligned} 24kj - 16k^2j &= 3(8kj) - 2k(8kj) \\ &= 8kj(3 - 2k) \end{aligned}$$

(j) $$\begin{aligned} -a^2b - b^2a &= -ab(a) - ab(b) \\ &= -ab(a + b) \end{aligned}$$

(k) $$\begin{aligned} 12k^2j + 24k^2j^2 &= 12k^2j + 12k^2j(2j) \\ &= 12k^2j(1 + 2j) \end{aligned}$$

(l) $$\begin{aligned} 72b^2q - 18b^3q^2 &= 18b^2q(4) + 18b^2q(-bq) \\ &= 18b^2q(4 - bq) \end{aligned}$$

(m) $$\begin{aligned} 4(y - 3) + k(3 - y) &= 4(y - 3) - k(y - 3) \\ &= (4 - k)(y - 3) \end{aligned}$$

(n) $$a(a - 1) - 5(a - 1) = (a - 5)(a - 1)$$

(o) $$\begin{aligned} bm(b + 4) - 6m(b + 4) &= (bm - 6m)(b + 4) \\ &= m(b - 6)(b + 4) \end{aligned}$$

(p) $$\begin{aligned} a^2(a + 7) + a(a + 7) &= a(a)(a + 7) + a(a + 7) \\ &= a[a(a + 7) + (a + 7)] \\ &= a[(a + 7)(a + 1)] \end{aligned}$$

(q) $$\begin{aligned} 3b(b - 4) - 7(4 - b) &= 3b(b - 4) + 7(b - 4) \\ &= (3b + 7)(b - 4) \end{aligned}$$

(r) $$\begin{aligned} a^2b^2c^2 - 1 &= (abc)^2 - (1)^2 \\ &= (abc - 1)(abc + 1) \end{aligned}$$ by difference of squares

Products

(a)

$$
\begin{aligned}
& (-2y^2 - 4y + 11)(5y - 12) \\
= \; & 5y(-2y^2 - 4y + 11) - 12(-2y^2 - 4y + 11) \\
= \; & -10y^3 - 20y^2 + 55y + 24y^2 - 48y - 132 \\
= \; & -10y^3 + (-20 + 24)y^2 + (55 + 48)y - 132 \\
= \; & -10y^3 + 4y^2 + 103y - 132
\end{aligned}
$$

(b)

$$
\begin{aligned}
& (-11y + 3)(-10y^2 - 7y - 9) \\
= \; & -11y(-10y^2 - 7y - 9) + 3(-10y^2 - 7y - 9) \\
= \; & 110y^3 + 77y^2 + 99y - 30y^2 - 21y - 27 \\
= \; & 110y^3 + (77 - 30)y^2 + (99 - 21)y - 27 \\
= \; & 110y^3 + 47y^2 + 78y - 27
\end{aligned}
$$

(c)

$$
\begin{aligned}
& (4y^2 + 12y + 10)(-9y^2 + 8y + 2) \\
= \; & 4y^2(-9y^2 + 8y + 2) + 12y(-9y^2 + 8y + 2) + 10(-9y^2 + 8y + 2) \\
= \; & -36y^4 + 32y^3 + 8y^2 - 108y^3 + 96y^2 + 24y - 90y^2 + 80y + 20 \\
= \; & -36y^4 + (32 - 108)y^3 + (8 + 96 - 90)y^2 + (24 + 80)y + 20 \\
= \; & -36y^4 - 76y^3 + 14y^2 + 104y + 20
\end{aligned}
$$

(d)

$$
\begin{aligned}
& (7y^2 - 6y - 8)(-2y + 2) \\
= \; & -2y(7y^2 - 6y - 8) + 2(7y^2 - 6y - 8) \\
= \; & -14y^3 + 12y^2 + 16y + 14y^2 - 12y - 16 \\
= \; & -14y^3 + (12 + 14)y^2 + (16 - 12)y - 16 \\
= \; & -14y^3 + 26y^2 + 4y - 16
\end{aligned}
$$

(e)

$$
\begin{aligned}
& (10y^5 + 3)(-2y^2 - 11y + 2) \\
= \; & 10y^5(-2y^2 - 11y + 2) + 3(-2y^2 - 11y + 2) \\
= \; & -20y^7 - 110y^6 + 20y^5 - 6y^2 - 33y + 6
\end{aligned}
$$

(f)

$$
\begin{aligned}
& (-12y - 3)(12y^2 - 11y + 3) \\
= \; & -12y(12y^2 - 11y + 3) - 3(12y^2 - 11y + 3) \\
= \; & -144y^3 + 132y^2 - 36y - 36y^2 + 33y - 9 \\
= \; & -144y^3 + (132 - 36)y^2 + (-36 + 33)y - 9 \\
= \; & -144y^3 + 96y^2 - 3y - 9
\end{aligned}
$$

(g) $(-10)(2y^2 + 8y + 3)$

(h)

$$
\begin{aligned}
& (2y^6 + 3y^5)(-5y - 12) \\
= \; & 2y^6(-5y - 12) + 3y^5(-5y - 12) \\
= \; & -10y^7 - 24y^6 - 15y^6 - 36y^5 \\
= \; & -10y^7 + (-24 - 15)y^6 - 36y^5 \\
= \; & -10y^7 - 39y^6 - 36y^5
\end{aligned}
$$

$$(6y^7 - 8y^2 + 7)(-4y - 3)(-6y^2 - 7y - 11)$$
$$= [-4y(6y^7 - 8y^2 + 7) - 3(6y^7 - 8y^2 + 7)](-6y^2 - 7y - 11)$$
$$= (-24y^8 - 18y^7 + 32y^3 + 24 - 28y - 21)(-6y^2 - 7y - 11)$$
$$= (-24y^8 - 18y^7 + 32y^3 - 28y + 3)(-6y^2 - 7y - 11)$$
$$= -24y^8(-6y^2 - 7y - 11) - 18y^7(-6y^2 - 7y - 11) + 32y^3(-6y^2 - 7y - 11) - 28y(-6y^2 - 7y - 11) + 3(-6y^2 - 7y - 11)$$
$$= 144y^{10} + 168y^9 + 264y^8 + 108y^9 + 126y^8 + 198y^7 - 192y^5 - 224y^4 - 352y^3 + 168y^3 + 196y^2 - 21y - 18y^2 - 21y - 33$$

(i) $= 144y^{10} + 276y^9 + 390y^8 + 198y^7 - 192y^5 - 224y^4 - 184y^3 + 178y^2 - 42y - 33$

(j)

$$(-9y^2 + 11y + 2)(8y^2 + 6y - 7)$$
$$= -9y^2(8y^2 + 6y - 7) + 11y(8y^2 + 6y - 7) + 2(8y^2 + 6y - 7)$$
$$= -72y^4 - 54y^3 + 63y^2 + 88y^3 + 66y^2 - 77y + 16y^2 + 12y - 14$$
$$= -72y^4 + 34y^3 + 145y^2 - 65y - 14$$

(k)

$$(8y^5 + 3y^4 + 2y^3)(5y + 10)(12y^2 + 6y + 6)$$
$$= [8y^5(5y + 10) + 3y^4(5y + 10) + 2y^3(5y + 10)](12y^2 + 6y + 6)$$
$$= (40y^6 + 80y^5 + 15y^5 + 30y^4 + 10y^4 + 20y^3)(12y^2 + 6y + 6)$$
$$= (40y^6 + 95y^5 + 40y^4 + 20y^3)(12y^2 + 6y + 6)$$
$$= 40y^6(12y^2 + 6y + 6) + 95y^5(12y^2 + 6y + 6) + 40y^4(12y^2 + 6y + 6) + 20y^3(12y^2 + 6y + 6)$$
$$= 480y^8 + 240y^7 + 240y^6 + 1140y^7 + 570y^6 + 570y^5 + 480y^6 + 240y^5 + 240y^4 + 240y^5 + 120y^4 + 120y^3$$
$$= 480y^8 + 1380y^7 + 1290y^6 + 1050y^5 + 360y^4 + 120y^3$$

(l)

$$(-7y + 11)(-12y + 3)$$
$$= -y^7(-12y + 3) + 11(-12y + 3)$$
$$= 84y^2 - 21y - 132y + 33$$
$$= 84y^2 - 153y + 33$$

(m)

$$(4y^3 + 5y^2 - 12y)(-12y - 2)(7y^2 - 9y + 12)$$
$$= [-12y(4y^3 + 5y^2 - 12y) - 2(4y^3 + 5y^2 - 12y)](7y^2 - 9y + 12)$$
$$= (-48y^4 - 60y^3 + 144y^2 - 8y^3 - 10y^2 + 24y)(7y^2 - 9y + 12)$$
$$= (-48y^4 - 68y^3 + 134y^2 + 24y)(7y^2 - 9y + 12)$$
$$= 7y^2(-48y^4 - 68y^3 + 134y^2 + 24y) - 9y(-48y^4 - 68y^3 + 134y^2 + 24y) + 12(-48y^4 - 68y^3 + 134y^2 + 24y)$$
$$= -336y^6 - 476y^5 + 938y^4 + 168y^3 + 432y^5 + 612y^4 - 1206y^3 - 216y^2 - 576y^4 - 816y^3 + 1608y^2 + 288y$$
$$= -336y^6 - 44y^5 + 974y^4 - 1854y^3 + 612y^4 + 1392y^2 + 288y$$

(n)

$$(7y + 3)(7y^2 + 3y + 10)$$
$$= 7y(7y^2 + 3y + 10) + 3(7y^2 + 3y + 10)$$
$$= 49y^3 + 21y^2 + 70y + 21y^2 + 9y + 30$$
$$= 49y^3 + 42y^2 + 79y + 30$$

(o)

$$9(8y^2 - 2y + 3) = 72y^2 - 18y + 27$$

(p)

$$\begin{aligned}
& (-12y + 12)(4y^2 - 11y - 11) \\
= \; & -12y(4y^2 - 11y - 11) + 12(4y^2 - 11y - 11) \\
= \; & -48y^3 + 132y^2 - 132y + 48y^2 - 132y + 132 \\
= \; & -48y^3 + 180y^2 - 264y + 132
\end{aligned}$$

(q)

$$\begin{aligned}
& (-64y^4 + 11y^2 + 3y)(10y + 4)(4y - 4) \\
= \; & [-64y^4(10y + 4) + 11y^2(10y + 4) + 3y(10y + 4)](4y - 4) \\
= \; & (-60y^5 - 24y^4 + 110y^3 + 44y^2 + 30y^2 + 12y)(4y - 4) \\
= \; & (-60y^5 - 24y^4 + 110y^3 + 74y^2 + 12y)(4y - 4) \\
= \; & 4y(-60y^5 - 24y^4 + 110y^3 + 74y^2 + 12y) - 4(-60y^5 - 24y^4 + 110y^3 + 74y^2 + 12y) \\
= \; & -240y^6 - 96y^5 + 440y^4 + 296y^3 + 48y^2 + 240y^5 + 96y^4 - 440y^3 - 296y^2 - 48y \\
= \; & -240y^6 + 144y^5 + 536y^4 - 144y^3 - 248y^2 - 48y
\end{aligned}$$

(r)

$$\begin{aligned}
& (-3y^6 - 6y^3)(11y - 6)(10y - 10) \\
= \; & [-3y^6(11y - 6) - 6y^3(11y - 6)](10y - 10) \\
= \; & (-33y^7 + 18y^6 - 66y^4 + 36y^3)(10y - 10) \\
= \; & 10y(-33y^7 + 18y^6 - 66y^4 + 36y^3) - 10(-33y^7 + 18y^6 - 66y^4 + 36y^3) \\
= \; & (-330y^8 + 180y^7 - 660y^5 + 360y^4 + 330y^7 - 180y^6 + 660y^4 - 360y^3) \\
= \; & (-330y^8 + 510y^7 - 180y^6 - 660y^5 + 1020y^4 - 360y^3)
\end{aligned}$$

(s)

$$\begin{aligned}
& (-11y^5 + 11y^4 + 11)(9y^3 - 7y^2 - 4y + 6) \\
= \; & 9y^3(-11y^5 + 11y^4 + 11) - 7y^2(-11y^5 + 11y^4 + 11) - 4y(-11y^5 + 11y^4 + 11) + 6(-11y^5 + 11y^4 + 11) \\
= \; & -99y^8 + 99y^7 + 99y^3 + 77y^7 - 77y^6 - 77y^2 + 44y^6 - 44y^5 - 44y - 66y^5 + 66y^4 + 66 \\
= \; & -99y^8 + (99 + 77)y^7 + (-77 + 44)y^6 + (-44 - 66)y^5 + 66y^4 + 99y^3 - 77y^2 - 44y + 66 \\
= \; & -99y^8 + 176y^7 - 33y^6 - 110y^5 + 66y^4 + 99y^3 - 77y^2 - 44y + 66
\end{aligned}$$

(t)

Factorising a Trinomial

1.
   (a) $x^2 + 8x + 15 = (x + 5)(x + 3)$
   (b) $x^2 + 10x + 24 = (x + 6)(x + 4)$
   (c) $x^2 + 9x + 8 = (x + 8)(x + 1)$
   (d) $x^2 + 9x + 14 = (x + 7)(x + 2)$
   (e) $x^2 + 15x + 36 = (x + 12)(x + 3)$
   (f) $x^2 + 12x + 36 = (x + 6)(x + 6)$

2.
   (a) $x^2 - 2x - 15 = (x + 5)(x - 3)$
   (b) $x^2 + 2x - 3 = (x + 3)(x - 1)$
   (c) $x^2 + 2x - 8 = (x + 4)(x - 2)$
   (d) $x^2 + x - 20 = (x + 5)(x - 4)$
   (e) $x^2 - x - 20 = (x - 5)(x + 4)$

3.

(a) $2x^2 + 11x + 5 = (2x + 1)(x + 5)$

(b) $3x^2 + 19x + 6 = (3x + 1)(x + 6)$

(c) $6x^2 + 7x + 2 = (3x + 2)(2x + 1)$

(d) $12x^2 + 8x + 1 = (6x + 1)(2x + 1)$

(e) $8x^2 + 6x + 1 = (4x + 1)(2x + 1)$

4.

(a) $3x^2 + 17x - 6 = (3x - 1)(x + 6)$

(b) $7x^2 - 6x - 1 = (7x + 1)(x - 1)$

(c) $8x^2 - 6x + 1 = (4x - 1)(2x - 1)$

(d) $2x^2 - 5x - 3 = (2x + 1)(x - 3)$

Factorisation by Grouping

1.
$$\begin{aligned} & 6x + a + 2ax + 3 \\ = \; & 3(2x + 1) + a(2x + 1) \\ = \; & (3 + a)(2x + 1) \end{aligned}$$

2.
$$\begin{aligned} & x^2 - 6x + 5x - 30 \\ = \; & x(x - 6) + 5(x - 6) \\ = \; & (x + 5)(x - 6) \end{aligned}$$

3.
$$\begin{aligned} & 5x + 10y - ax - 2ay \\ = \; & 5(x + 2y) - a(x + 2y) \\ = \; & (5 - a)(x + 2y) \end{aligned}$$

4.
$$\begin{aligned} & a^2 - 2a - ax + 2x \\ = \; & a(a - 2) - x(a - 2) \\ = \; & (a - x)(a - 2) \end{aligned}$$

5.
$$\begin{aligned} & 5xy - 3y + 10x - 6 \\ = \; & y(5x - 3) + 2(5x - 3) \\ = \; & (y + 2)(5x - 3) \end{aligned}$$

Simplification of Fractions

1.

(a) $\frac{3a}{15}=\frac{3a}{3(5)}=\frac{a}{5}$

(b) $\frac{2a+10}{4}=\frac{2(a+5)}{2(2)}=\frac{a+5}{2}$

(c) $\frac{5a+20}{a+4}=\frac{5(a+4)}{a+4}=5 \text{ provided } a\neq -4$

(d) $\frac{a^2-4a}{a-4}=\frac{a(a-4)}{a-4}=a \text{ provided } a\neq 4$

(e) $\frac{3a^2-9a}{2a-6}=\frac{3a(a-3)}{2(a-3)}=\frac{3a}{2} \text{ provided } a\neq 3$

(f) $\frac{9a+27}{9a+18}=\frac{9(a+3)}{9(a+2)}=\frac{a+3}{a+2}$

(g) $\frac{6ab+2a}{2b}=\frac{2a(3b+1)}{2b}=\frac{a(3b+1)}{b}$

(h) $\frac{16x^2y-8xy}{12x-6}=\frac{8xy(2x-1)}{6(2x-1)}=\frac{2(4)xy}{2(3)}=\frac{4xy}{3} \text{ provided } x\neq \frac{1}{2}$

(i) $\frac{4xyp-8xp}{12xy}=\frac{4xp(y-2)}{3(4)xy}=\frac{p(y-2)}{3y}$

$$\frac{3a+9}{14}\div\frac{a+3}{7a+21}$$
$$=\frac{3(a+3)}{2(7)}\times\frac{7(a+3)}{a+3}$$
(j) $$=\frac{3(a+3)}{2}$$

$$\frac{a^2-5a}{2a+10}\div\frac{4a}{3a+15}$$
$$=\frac{a(a-5)}{2(a+5)}\times\frac{3(a+5)}{4a}$$
(k) $$=\frac{3(a-5)}{8}$$

$$\frac{3xp+4p}{8p}\div\frac{3x+4}{12p^2}$$
$$=\frac{p(3x+4)}{2(4)p}\times\frac{3(4)p^2}{(3x+4)}$$
(l) $$=\frac{3p^2}{2}$$

$$\frac{16}{2xp+4x}\div\frac{12}{6x^2+8x}$$
$$=\frac{4(4)}{2x(p+2)}\times\frac{2x(3x+4)}{3(4)}$$
(m) $$=\frac{4(3x+4)}{3(p+2)}$$

$$\frac{24a-8}{12}\div\frac{9a-3}{6}$$
$$=\frac{24a-8}{12}\times\frac{6}{9a-3}$$
$$=\frac{2(4)(3a-1)}{3(4)}\times\frac{2(3)}{3(3a-1)}$$
(n) $$=\frac{4}{3} \text{ provided } a\neq\frac{1}{3}$$

$$\frac{a^2+2a}{5}\div\frac{2a+4}{20}$$
$$=\frac{a^2+2a}{5}\times\frac{20}{2a+4}$$
$$=\frac{a(a+2)}{5}\times\frac{4(5)}{2(a+2)}$$
(o) $$=2a \text{ provided } a\neq -2$$

$$\frac{p^2+pq}{7p}\div\frac{8p+8q}{21q}$$
$$=\frac{p^2+pq}{7p}\times\frac{21q}{8p+8q}$$
$$=\frac{p(p+q)}{7p}\times\frac{3(7)q}{8(p+q)}$$
(p) $$=\frac{3q}{8} \text{ provided } p\neq -q$$

$$\frac{5ab-15b}{4a-12}\div\frac{6b^2}{a+b}$$
$$=\frac{5ab-15b}{4a-12}\times\frac{a+b}{6b^2}$$
$$=\frac{5b(a-3)}{4(a-3)}\times\frac{(a+b)}{6b(b)}$$
(q) $$=\frac{5(a+b)}{24b} \text{ provided } a\neq 3$$

$$\frac{f^2a-fa^2}{f-a}$$
$$=\frac{fa(f-a)}{f-a}$$
(r) $$=fa \text{ provided } f\neq a$$

2. $\frac{x^2-1}{3}\times\frac{1}{x-1}-\frac{1}{2}$

$= \frac{(x-1)(x+1)}{3(x-1)}-\frac{1}{2}$

$= \frac{1}{3}(x+1)-\frac{1}{2}$

$= \frac{1}{3}x-\frac{1}{3}-\frac{1}{2}$

$= \frac{1}{3}x-\frac{1}{6}$

End of Chapter Exercises

1.
(a) $a^2-9=(a-3)(a+3)$
(b) $m^2-36=(m+6)(m-6)$
(c) $9b^2-81=(3b-9)(3b+9)$
(d) $16b^2-25a^2=(4b+5a)(4b-5a)$
(e) $m^2-\left(\frac{1}{9}\right)=\left(m+\frac{1}{3}\right)\left(m-\frac{1}{3}\right)$
(f) $5-5a^2b^6=5\left(1-a^2b^6\right)=5\left(1-ab^3\right)\left(1+ab^3\right)$
(g) $16ba^4-81b=b\left(16a^4-81\right)=b\left(4a^2+9\right)\left(4a^2-9\right)=b\left(4a^2+9\right)(2a+3)(2a-3)$
(h) $a^2-10a+25=(a-5)(a-5)$
(i) $16b^2+56b+49=(4b+7)(4b+7)$
(j) $2a^2-12ab+18b^2=(2a-6b)(a-3b)$
(k) $-4b^4-144b^8+48b^5=-4b^2\left(36b^6-12b^3+1\right)=-4b^2\left(6b^3-1\right)\left(6b^3-1\right)$

2.
a) $\left(4-x^2\right)\left(4+x^2\right)=(2-x)(2+x)\left(4+x^2\right)$
b) $\left(7x^2-14x\right)+(7xy-14y)=7x(x-2)+7y(x-2)$
$=(x-2)(7x+7y)$
$=7(x-2)(x+y)$
c) $(y-10)(y+3)$
d) $(1-x)-x^2(1-x)=(1-x)\left(1-x^2\right)=(1-x)(1-x)(1+x)$
e) $-3(1-p)(1+p)+(p+1)=(p+1)[-3(1-p)+1]=(p-1)(-3p-2)$

3.
a) $a^2-4a+4-a^2-4a=-8a+4$
b) $125a^3-64b^3$
c) $\left(4m^2-9\right)\left(4m^2+9\right)=16m^4-81$
d) $(a+2b)^2-c^2=a^2+4ab+4b^2-c^2$

4.
a) $\frac{(p-q)(p+q)}{p}\times\frac{p(p-q)}{p+q}=(p-q)^2=p^2-2pq+q^2$
b) $\frac{12+3x^2-4x^2}{6x}=\frac{12-x^2}{6x}$

5.
$$(2x-1)^2-(x-3)^2$$
$$=(2x-1)(2x-1)-(x-3)(x-3)$$
$$=4x^2-2x-2x+1-\left(x^2-3x-3x+9\right)$$
$$=(4-1)x^2+(-4+6)x+(1-9)$$
$$=3x^2+2x-8$$
$$=(3x-4)(x+2)$$

6. Suppose A must be added to $x^2 - x + 4$ to make it equal to $(x+2)^2$.

$$
\begin{aligned}
(x^2 - x + 4) + A &= (x+2)^2 \\
A &= (x+2)(x+2) - (x^2 - x + 4) \\
&= x^2 + 2x + 2x + 4 - x^2 + x - 4 \\
&= (1-1)x^2 + (2+2+1)x + (4-4) \\
&= 5x
\end{aligned}
$$

Therefore $5x$ must be added to $x^2 - x + 4$ to make it equal to $(x+2)^2$.

Chapter 10 - Equations and Inequalities

http://cnx.org/content/m38372/latest/?collection=col11306/latest

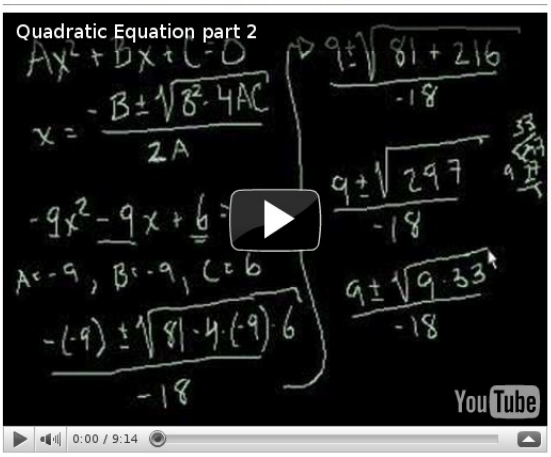
Videos

Khan academy video on equations - 1

This Khan Academy video on Equations 3 covers some examples of solving linear equations. This video shows how to group similar terms and move all the variables to one side of the equals sign and the numbers to the other.

This video can be downloaded at: http://www.fhsst.org/lCx

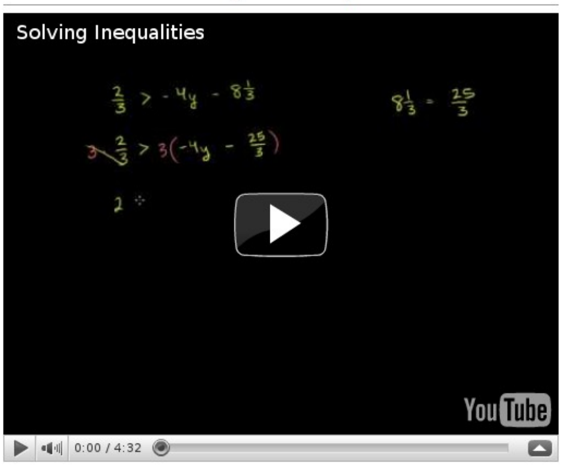
Khan academy video on equations - 3



This Khan Academy video on Quadratic Equations covers how to work with the quadratic equation. This video covers some examples of using the quadratic equation.

This video can be downloaded at: http://www.fhsst.org/lC1

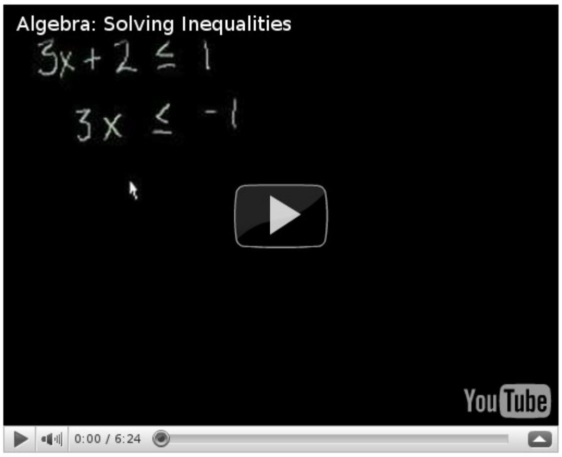
Khan academy video on inequalities - 1



This Khan Academy video on Solving Inequalities covers a simple example of how to solve a linear inequality. This video shows how to extend the concepts used in solving equations to solving inequalities.

This video can be downloaded at: http://www.fhsst.org/lCC

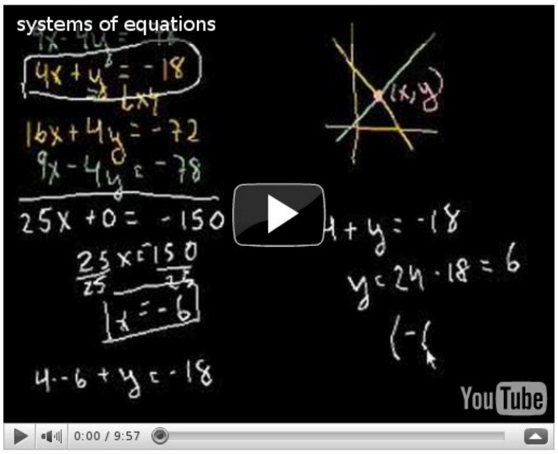
Khan academy video on inequalities - 2



This Khan Academy video on Solving Inequalities covers some more examples of how to solve linear inequalities. This video also covers representing inequalities on a number line.

This video can be downloaded at: http://www.fhsst.org/lCa

Khan academy video on simultaneous equations - 1



This Khan Academy video on Systems of Equations covers the concept of simultaneous equations and how to solve simultaneous equations. This video covers the algebraic solution to simultaneous equations.

This video can be downloaded at: http://www.fhsst.org/lCr

Solutions

Solving Linear Equations

1. $$\begin{aligned} 2y &= 7 \quad 3 \\ 2y &= 10 \\ y &= 5 \end{aligned}$$

2. $y = \frac{0}{-3} = 0$

3. $y = \frac{16}{4} = 4$

4. $$\begin{aligned} 12y &= 144 - 0 \\ y &= \frac{144}{12} = 12 \end{aligned}$$

5. $$\begin{aligned} 5y &= 62 - 7 \\ 5y &= 55 \\ y &= 11 \end{aligned}$$

6. $$\begin{aligned} \tfrac{3}{4} &= 55 - 5x \\ 3 &= 220 - 20x \\ 20x &= 117 \\ x &= \tfrac{117}{20} \end{aligned}$$

7. $$\begin{aligned} 5x - 3x &= 45 \\ 2x &= 45 \\ x &= \tfrac{45}{2} \end{aligned}$$

8. $$\begin{aligned} 23x - 2x &= 6 \quad 12 \\ 21x &= 18 \\ x &= \tfrac{18}{21} \end{aligned}$$

9. $$\begin{aligned} -6x \quad 34x - 2x &= -24 - 64 - 12 \\ 26x &= -100 \\ x &= \tfrac{-100}{26} \end{aligned}$$

10. $$\begin{aligned} 6x \quad 3x &= 4 - 10x \quad 15 \\ 9x \quad 10x &= 4 - 15 \\ 19x &= -11 \\ x &= \tfrac{-11}{19} \end{aligned}$$

11. $$\begin{aligned} 18 - 9 &= p \quad 2p \\ 9 &= 3p \\ p &= 3 \end{aligned}$$

12. $$\begin{aligned} 96 &= 16p \\ p &= 6 \end{aligned}$$

13. $$p = 8$$

14. $$\begin{aligned} 16 \quad p &= 13p - 1 \\ 13p - p &= 16 \quad 1 \\ 12p &= 17 \\ p &= \tfrac{17}{12} \end{aligned}$$

15. $$\begin{aligned} 8p - 4p &= 6 \quad 2 \\ 4p &= 8 \\ p &= 2 \end{aligned}$$

16. $$\begin{aligned} 3f &= 20 \\ f &= \tfrac{20}{3} \end{aligned}$$

17. $$\begin{aligned} 4f - 3f &= 16 \quad 10 \\ f &= 26 \end{aligned}$$

18. $$\begin{aligned} 10f \quad 5 &= -5f \quad 80 \\ 10f \quad 5f &= 80 - 5 \\ 15f &= 75 \\ f &= 5 \end{aligned}$$

19. $$\begin{aligned} 8f - 32 &= 5f - 20 \\ 8f - 5f &= -20 \quad 32 \\ 3f &= 12 \\ f &= 4 \end{aligned}$$

20. $$\begin{aligned} 6 &= 6f \quad 42 \quad 5f \\ 11f &= -36 \\ f &= \tfrac{-36}{11} \end{aligned}$$

Solving Quadratic Equations

1. a) 3x = -2 or 3x = 4
$x = \frac{-2}{3}$ $\quad x = \frac{4}{3}$

2. b) 5x = 9
$x = \frac{9}{5}$or x = -6

3. c) $x = \frac{-3}{2}$ or $x = \frac{3}{2}$

4. d) $x = \frac{-1}{2}$ or $x = \frac{9}{2}$

5. e) $x = \frac{3}{2}$ or $x = \frac{-2}{3}$

6. a) $x \quad 20 \quad 25x \quad = 0$
$25x = -20$
x = 0 or $x = \frac{-20}{25} = \frac{-4}{5}$

7. b) $4x \quad 11 \qquad x - 7 \quad = 0$
$4x = -11$
x = 7 or $x = \frac{-11}{4}$

8. c) $2x \quad 3 \qquad x - 4 \quad = 0$
x = 4 or $x = \frac{-3}{2}$

9. $$\begin{aligned} -15x^2 \quad 58 - 48 &= 0 \\ 15x^2 - 58 \quad 48 &= 0 \\ 3x - 8 \qquad 5x - 6 \quad &= 0 \end{aligned}$$
$x = \frac{8}{3}$or $x = \frac{6}{5}$

10. $$\begin{aligned} 6y &= y^2 - 9y \quad 44 \\ & y^2 - 15y \quad 44 \\ y - 4 \qquad y - 11 \quad &= 0 \end{aligned}$$
y = 4 or y = 11

11.
$$x^2 - 4x \quad 4 = 0$$
$$x - 2 \quad\quad x - 2 \quad = 0$$
$$x = 2$$

12.
$$4x^2 \quad x^2 - 5x - 4x \quad 3 \quad 6 = 0$$
$$5x^2 - 9x \quad 9 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{- \quad -9 \quad \pm \sqrt{\quad -9 \quad ^2 - 4 \quad 5 \quad\quad 9}}{2} \quad 5$$
$$x = \frac{9 \pm \sqrt{81 - 180}}{10}$$
$$x = \frac{9 \pm \sqrt{-99}}{10}$$

no real solution or roots are imaginary

13.
$$x^2 - 3x = 0$$
$$x \quad x - 3 \quad = 0$$

x = 0 or x = 3

14.
$$3x^2 \quad 10x - 25 = 0$$
$$3x - 5 \quad\quad x \quad 5 \quad = 0$$
x = -5 or $x = \frac{5}{3}$

15.
$$x^2 - x \quad 3 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{- \quad -1 \quad \pm \sqrt{\quad -1 \quad ^2 - 4 \quad 3 \quad\quad 1}}{2}$$
$$x = \frac{1 \pm \sqrt{-11}}{2}$$

The roots are imaginary

16.
$$x - 2 \quad\quad x - 2 \quad = 0$$
$$x = 2$$

17.
$$x^2 - 6x - 7 = 0$$
$$x \quad 1 \quad\quad x - 7 \quad = 0$$

x = -1 or x = 7

18.
$$14x^2 \quad 5x - 6 = 0$$
$$7x \quad 6 \quad\quad 2x - 1 \quad = 0$$
$x = \frac{-6}{7}$ or $x = \frac{1}{2}$

19.
$$2x^2 - 2x - 12 = 0$$
$$x^2 - x - 6 = 0$$
$$x \quad 2 \quad\quad x - 3 \quad = 0$$

x = -2 or x = 3

20.

$$3x^2 - x^2 \quad 2x \quad x - 6 - 2 = 0$$
$$2x^2 \quad 3x - 8 = 0$$
$$x^2 \quad \tfrac{3}{2}x - 4 = 0$$
$$x = \frac{-\frac{3}{2} \pm \sqrt{\quad \frac{3}{2} \quad ^2 - 4 \quad 1 \quad\quad -8}}{2}$$
$$x = -\tfrac{3}{2} \pm \frac{\sqrt{\frac{9}{4} \quad 32}}{2}$$
$$x = \frac{\frac{-3}{2} \pm 5,87}{2}$$

$x = 2,185$ or $x = -3,72$

Solving Exponential Equations

1. a) $x \quad 5 = 5$
   $x = 0$

   b) $2x \quad 1 = 3$
   $2x = 2$
   $x = 1$

   c) $2x \quad 2 = 3$
   $2x = 1$
   $x = \frac{1}{2}$

   d) $5 - x = 12$
   $x = -7$

   e) $2^{6(x+1)} = 2^{4(2x+5)}$
   $6x + 6 = 8x + 20$
   $2x = -14$
   $x = -7$

   f) $5^{3(x)} = 5^1$
   $3x = 1$
   $x = \frac{1}{3}$

2. $3^{9x-2} = 3^3$
   $9x - 2 = 3$
   $9x = 5$
   $x = \frac{5}{9}$

3. $3^{4(x+2)} = 3^{3(x+4)}$
   $4x + 8 = 3x + 12$
   $x = 4$

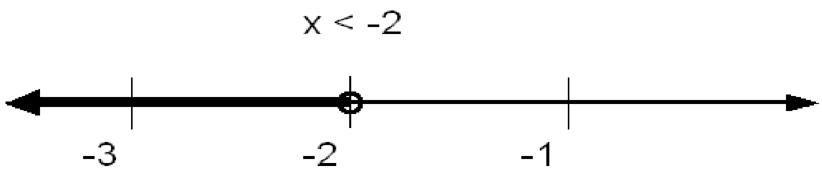
4. $f(t) = 2^t$
   $2^t = 128$
   $2^t = 2^7$
   $t = 7$

5.
$$5^{2(1-2x)}=5^4$$
$$2-4x=4$$
$$4x=-2$$
$$x=\frac{-1}{2}$$

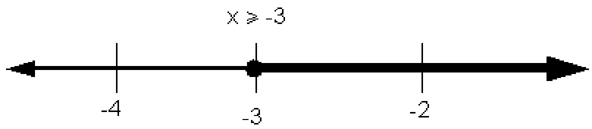
6. b) Note that $3^0=1$
$$3^{3x}\times 3^{2(x-2)}=3^0$$
$$3x+2x-4=0$$
$$5x=4$$
$$x=\frac{4}{5}$$

Linear Inequalities

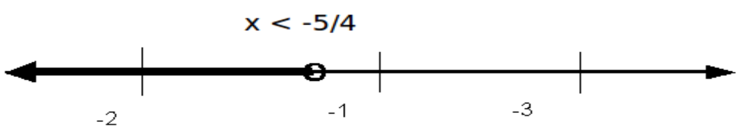
1. a)
$$3x-5x>8-4$$
$$-2x>4$$
$$2x<-4$$
$$x<-2$$



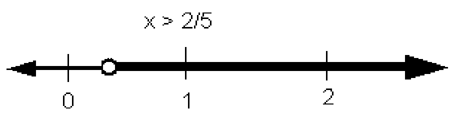
b)
$$3x-3-2\le 6x+4$$
$$3x-6x\le 4+5$$
$$-3x\le 9$$
$$3x\ge -9$$
$$x\ge -3$$



c)
$$2(x-7)>3(2x-3)$$
$$2x-14>6x-9$$
$$-4x>5$$
$$x<\frac{-5}{4}$$



d)
$$-4x+4<x+2$$
$$-5x<-2$$
$$x>\frac{2}{5}$$

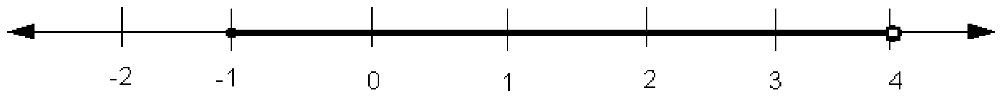


e)
$$3x+2(x-1)\ge 5x-2$$
$$3x+2x-2\ge 5x-2$$
$$5x-5x\ge -2+2$$

true for all real values of x

2 a) $-2+1 \le x-1+1 < 3+1$

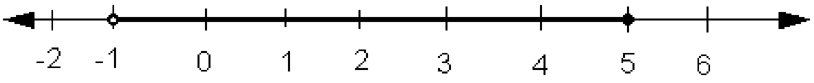
$-1 \le x < 4$



b) $-5+3 < 2x-3+3 \le 7+3$

$-2 < 2x \le 10$

$-1 < x \le 5$

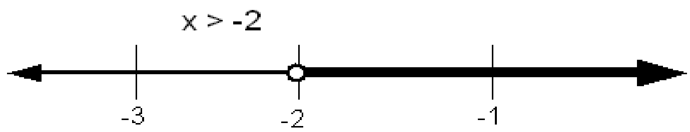


3. $21x+14-10x+15 > 7$

$11x > 7-14-15$

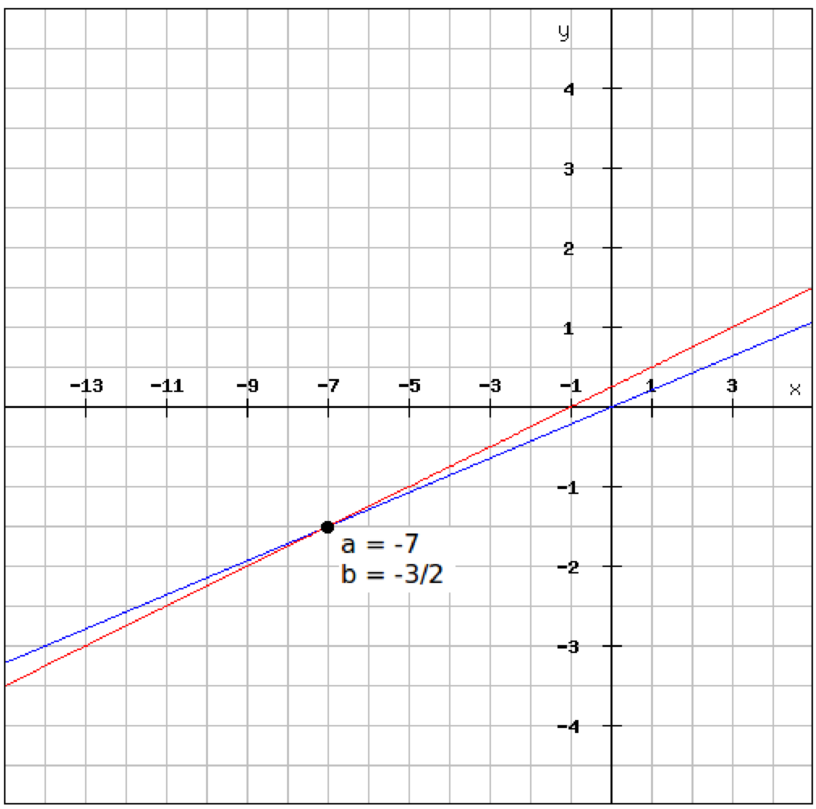
$11x > -22$

$x > -2$



Simultaneous Equations

1.



(Note that a = x and b = y)

$$3a = 14b$$
$$a = \frac{14b}{3}$$

$$\frac{14b}{3} - 4b \quad 1 = 0$$
$$14b - 12b \quad 3 = 0$$
$$2b = -3$$
$$b = \frac{-3}{2}$$

$$a = \frac{14\left(\frac{-3}{2}\right)}{3}$$
$$a = -7$$

$$15c = 132 - 11d$$
$$c = \frac{132 - 11d}{15}$$

2.

$$2\left(\frac{132 - 11d}{15}\right) \quad 3d - 59 = 0$$
$$\frac{264 - 22d}{15} = 59 - 3d$$
$$264 - 22d = 885 - 45d$$
$$23d = 621$$
$$d = 27$$

$$c = \frac{132 - 11 \quad 27}{15}$$
$$c = -11$$

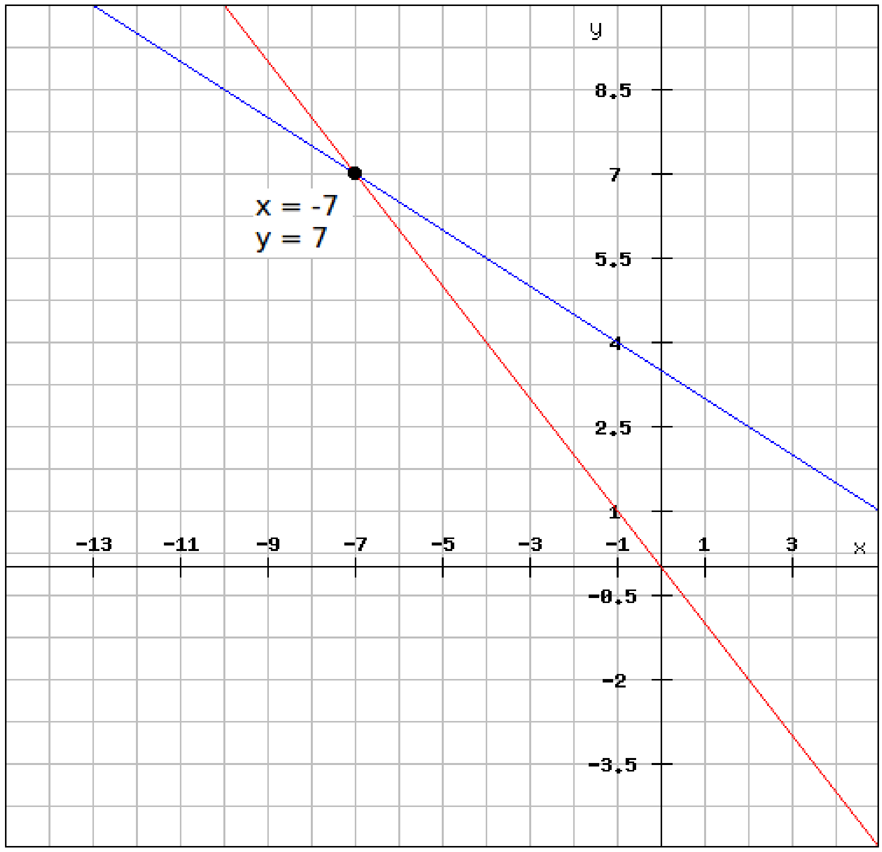
3.

$$6e \quad 6 - f = 0$$
$$f = 6e - 6$$

$$e - 4 \quad 6e - 6 \qquad 47 = 0$$
$$e - 24e \quad 24 \quad 47 = 0$$
$$23e = 71$$
$$e = \frac{71}{23}$$

$$f = 6 \quad \frac{71}{23} \quad - 6$$
$$f = \frac{426 - 138}{23}$$
$$f = \frac{288}{23}$$

4.



x = -7 and y = 7

Solving Literal Equations

1. $\frac{v-u}{a}=t$

2. $x(a-b)=c$

   $x=\frac{c}{(a-b)}$

3. $\frac{bx}{1}[\frac{1}{b}+\frac{2b}{x}]=[\frac{2}{1}]\frac{bx}{1}$

   $x+2b(b)=2bx$

   $x-2bx=-2b^2$

   $x(1-2b)=-2b^2$

   $x=\frac{-2b^2}{(1-2b)}$

Mathematical Models

1. 1l contains 69% of salt, so it must contain 690ml of salt.

$$\text{total volume } = 2 \times 690\text{ml} = 1380\text{ml} = 1{,}38\text{l}$$

Water to be added = 1 380 – 1 000 = 380ml

% added = 38%

2. Let the rectangle length = x and the rectangle width = 17 – x.

The diagonal is:

$$\text{diagonal } = \tfrac{1}{2} \times w \times l$$

$$= \tfrac{1}{2} \quad 17 - x \qquad x$$

$$x \quad 8 = \tfrac{1}{2}x^2 - 17x$$

$$2x \quad 16 = x^2 - 17x$$

$$x^2 - 19x - 16 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{19 \pm \sqrt{\quad 19 \quad ^2 - 4 \quad 1 \qquad -16 \quad}}{2}$$

$$x = \frac{19 \pm \sqrt{297}}{2}$$

x = 0,883 or x = 18,117

3. Let the unknown number = x. Then:

$$27 \quad 12 = x \quad 73$$

$$39 = x \quad 73$$

$$x = 39 - 73$$

$$x = -34$$

The unknown number is -34.

4. Let the smaller angle = x and then the other angle = 2x. The angles in a triangle add up to 180 and we know that the third angle is 90. So:

$$x \quad 2x \quad 90 = 180$$

$$3x = 90$$

$$x = 30$$

5. We first need to work out George's costs:

costs = 150 + 50 + 5 = 205

George receives (i.e. his income) R 400 for each cake sold.

His profit is therefore:

$$\text{profit } = \text{ income } - \text{ costs}$$

$$= 400 - 205 = 195$$

As a percentage this is:

$$\tfrac{195}{205} = 0{,}9512 = 95{,}12\,\%$$

6. Let the number = x. The equation that expresses the information given is:

$$4x \quad 7 = x^2 - 15$$

Solving this we get:

$$x^2 - 4x - 22 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{\quad -4 \quad ^2 - 4 \quad 1 \qquad -22 \quad}}{2}$$

$$x = \frac{4 \pm \sqrt{104}}{2}$$

x = 7,099 or x = -3,099

7. Let the width = x and the length = x + 2

Perimeter = P = 2(width) + 2(length)

$$P = 2 \quad x \qquad 2 \quad x \quad 2$$
$$20 = 2x \quad 2x \quad 2$$
$$16 = 4x$$
$$x = 4$$

So the width is 4cm and the length is 6cm.

End of Chapter Exercises

1. $$x - 2 \qquad x - 1 \quad = 0$$

The roots and 2 and 1. (Alternatively we could say x = 2 or x = 1)

2. $$x^2 \quad x - 6 = 0$$
$$x \quad 3 \qquad x - 2 \quad = 0$$

x = 2 or x = -3

3. We set y = 0 and solve for x:

$$0 = 2x^2 - 5x - 18$$
$$2x \quad 9 \qquad x - 2 \quad = 0$$

x = 2 or $x = \frac{-9}{2}$

4. Let Manuel have x CDs. Then Pedro has x – 5 CDs. And Bob had 2x CDs.

If we add up the total number of CDs we get:

$$x \qquad x - 5 \qquad 2x = 63$$
$$x \quad x - 5 \quad 2x = 63$$
$$4x = 68$$
$$x = 17$$

So Manuel has 17 CDs, Pedro has 12 CDs and Bob has 34 CDs.

5. Let the number = x.

Then:

$$\tfrac{7}{8}x \quad 5 = \tfrac{1}{3}x$$
$$21x \quad 120 = 8x$$
$$13x = -120$$
$$x = \tfrac{-120}{13}$$

6. Let x be the distance to the telephone.

$$900 = 4x \quad 5x$$
$$900 = 9x$$
$$x = 100m$$

7. $$\tfrac{x - 42}{3} \quad \tfrac{56 - x}{4}$$
$$4x - 168 \quad 168 - 3x$$
$$7x \quad 336$$
$$x \quad 48$$

a) $48, \infty$

b) $49,50$

8. $3 \quad 1 - a \quad - 2 \quad 2 - a \quad\quad 6$

$$3 - 3a - 4 \quad 2a \quad 6$$
$$-a \quad 7$$
$$a \quad -7$$

9. $x \neq 0$

$$x^2 - x = 42$$
$$x^2 - x - 42 = 0$$
$$x - 7 \quad\quad x \quad 6 \quad\quad = 0$$

x = 7 or x = -6

10. $3y = 13 - 7x$

$$y = \frac{13 - 7x}{3}$$
$$2x - 3\frac{13 - 7x}{3} = -4$$
$$2x - 13 \quad 7x = -4$$
$$9x = 9$$
$$x = 1$$
$$y = \frac{13 - 7 \quad 1}{3}$$
$$y = \frac{6}{3} = 2$$

Chapter 11 - Average Gradient

http://cnx.org/content/m38352/latest/?collection=col11306/latest

Solutions

End of Chapter Exercises

1. $t_1=2$

   $t_2=3$

   $$\begin{aligned} d_1 &= 2t_1^2+1 \\ &= 2(2)^2+1 \\ &= 2(4)+1 \\ &= 9 \\ d_2 &= 2t_2^2+1 \\ &= 2(3)^2+1 \\ &= 2(9)+1 \\ &= 19 \\ \frac{d_2-d_1}{t_2-t_1} &= \frac{19-9}{3-2} \\ &= \frac{10}{1} \\ &= 10 \end{aligned}$$

   The average speed of the object between $t=2$ and $t=3$ on the curve $d=2t^2+1$ is 10.

2. Answer

   $x_1=1$

   $x_2=4$

   $$\begin{aligned} f(x_1) &= x_1^3-6x_1 \\ &= (1)^3-6(1) \\ &= 1-6 \\ &= -5 \\ f(x_2) &= x_2^3-6x_2 \\ &= (4)^3-6(4) \\ &= 64-24 \\ &= 40 \\ \frac{f(x_2)-f(x_1)}{x_2-x_1} &= \frac{40-(-5)}{4-1} \\ &= \frac{40+5}{3} \\ &= \frac{45}{3} \\ &= 15 \end{aligned}$$

   The average gradient between $x=1$ and $x=4$ on the curve $f(x)=x^3-6x$ is 15.

3. a) When x = 2, we find:

$$y = x^2 + 3$$
$$y = (2)^2 + 3$$
$$y = 7$$

When x = 3, we find:

$$y = x^2 + 3$$
$$y = (3)^2 + 3$$
$$y = 12$$

So the average gradient is:

$$\text{average gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{12 - 7}{3 - 2}$$
$$= 5$$

b) When x = 2, we find:

$$y = \frac{4}{x} + 1$$
$$y = \frac{4}{2} + 1$$
$$y = 3$$

When x = 3, we find:

$$y = \frac{4}{x} + 1$$
$$y = \frac{4}{3} + 1$$
$$y = 2{,}33$$

So the average gradient is:

$$\text{average gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2{,}33 - 3}{3 - 2}$$
$$= -0{,}67$$

c) When x = 2, we find:

$$y = 2^x + 3$$
$$y = (2)^2 + 3$$
$$y = 7$$

When x = 3, we find:

$$y = 2^x + 3$$
$$y = (2)^x + 3$$
$$y = 11$$

So the average gradient is:

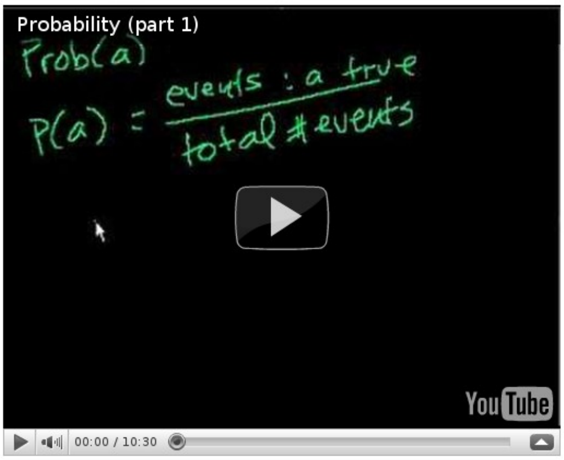
$$\text{average gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{11 - 7}{3 - 2}$$
$$= 4$$

Chapter 12 - Probability

http://cnx.org/content/m38378/latest/?collection=col11306/latest

Videos

Khan academy video on probability



This Khan Academy video on Probability covers an introduction to the concept of probability and then covers several examples of probability and how to work out probability.

This video can be downloaded at: http://www.fhsst.org/lCX

Solutions

Probability Models

1. A 6/12 = 1/2
   B (3 + 1)/12 = 1/3
   C 1 - (2/12) = 5/6
   D 1 - (2 + 6)/12 = 4/12 = 1/3

2. A: 1/52 (only one in the deck)
   B: 1/2 (half the cards are red, half are black)
   C: 3/13 (for each suite of 13 cards, there are three picture cards: J, Q, K)
   D: 4/52 = 1/13 (four aces in the deck)
   E: 3/13 (for each suite of 13 cards, there are three cards less than 4: A, 2 and 3)

3. There are 50 cards. They are all even.
   All even numbers that are also multiples of 5 are multiples of 10 (10,20,...,100).
   There are 10 of them.
   Therefore, the probability is 10/50 = 1/5.

Probability Identities

1. If he fired just one shot, the probability that it would miss is 0,3.

   So the probability that all five shots miss is simply the probability that each shot misses multiplied :

   $P(\text{ all miss }) = 0{,}3 \times 0{,}3 \times 0{,}3 \times 0{,}3 \times 0{,}3 = 0{,}00243$

2. The probability that all arrows hit the bullseye is:

   $P(\text{ all hit }) = 0{,}4 \times 0{,}4 \times 0{,}4 = 0{,}064$

3. a) The probability that a tail is tossed is 0,5.

   The probability of a 9 being rolled is: $\frac{1}{6} = 0{,}167$

   The probability that a tail is tossed and a 9 rolled is:

   $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12} = 0{,}083$

   b) This is the same as in part ii. The probability that a head is tossed is 0,5 and the probability of getting a 3 is 0,167. So the probability that a head is tossed and a 3 rolled is 0,083.

4. a) The probability that all four pass is simply the individual probabilities multiplied together:

   $P(\text{ 4 pass }) = 0{,}8 \times 0{,}5 \times 0{,}6 \times 0{,}9 = 0{,}216$

   b) The probability that all four fail is the probability that each one fails multiplied together:

   $P(\text{ 4 fail }) = 0{,}2 \times 0{,}5 \times 0{,}4 \times 0{,}1 = 0{,}004$

5. The probability of getting an ace is $\frac{4}{52} = 0{,}077$. The probability of getting a black card is 0,5. (all cards are either red or black and there is an equal number of red and black cards). The probability that the card picked will be either an ace or a black card is:

   $$\begin{aligned} P(\text{ace} \cup \text{ black }) &= P(\text{ace}) + P(\text{ black }) - P(\text{ace} \cap \text{ black }) \\ &= 0{,}077 + 0{,}5 - 0{,}5 \times 0{,}077 \\ &= 0{,}5385 \end{aligned}$$

Mutually Exclusive Events

1. Before we answer the questions we first work out how many blocks there are in total. This gives us the sample space.

   $n(S) = 24 + 32 + 41 + 19 = 116$

   a) The probability that a block is purple is:

   $P(\text{Purple}) = \frac{n(E)}{n(S)}$

   $P(\text{Purple}) = \frac{24}{116}$

   $P(\text{Purple}) = 0{,}21$

   b) The probability that a block is either purple or white is:

   $$\begin{aligned} P(\text{ purple } \cup \text{ white }) &= P(\text{ purple }) + P(\text{ white }) - P(\text{ purple } \cap \text{ white }) \\ &= \frac{24}{116} + \frac{41}{116} - \frac{24}{116} \times \frac{41}{116} \\ &= 0{,}64 \end{aligned}$$

   c) Since one block cannot be two colours the probability of this event is 0.

   d) We first work out the probability that a block is orange:

   $P(\text{orange}) = \frac{32}{116} = 0{,}28$

   The probability that a block is not orange is:

   $P(\text{ not orange }) = 1 - 0{,}28 = 0{,}72$

2. We calculate the total number of pupils at the school:

$6+2+5+7+4+6=30$

a) The total number of female children is $6+5+4=15$.

The probability of a randomly selected child being female is:

$P(\text{female})=\frac{n(E)}{n(S)}$

$P(\text{female})=\frac{15}{30}$

$P(\text{female})=0{,}5$

b) The probability of a randomly selected child being a 4 year old male is:

$P(4\text{male})=\frac{7}{30}=0{,}23$

c) There are $6+2+5+7=20$children aged 3 or 4.

The probability of a randomly selected child being either 3 or 4 is:

$\frac{20}{30}=0{,}67$

d) A child cannot be both 3 and 4, so the probability is 0.

e) This is the same as a randomly selected child being either 3 or 4 and so is 0,67.

f) The probability of a child being either 3 or female is:

$$\begin{aligned}P(3\cup\text{female})&=P(3)+P(\text{female})-P(3\cap\text{female})\\&=\frac{10}{30}+0{,}5-\frac{10}{30}\times\frac{15}{30}\\&=0{,}67\end{aligned}$$

3.

a) The set of all discs ending with 5 is: {5, 15, 25, 35, 45, 55, 65, 75, 85}. This has 9 elements.

The probability of drawing a disc that ends with 5 is:

$P(5)=\frac{n(E)}{n(S)}$

$P(5)=\frac{9}{85}$

$P(5)=0{,}11$

b) The set of all discs that can be multiplied by 3 is: {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84}. This has 28 elements.

The probability of drawing a disc that can be multiplied by 3 is:

$P(3_m)=\frac{28}{85}=0{,}33$

c) The set of all discs that can be multiplied by 6 is: {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84}. This set has 14 elements.

The probability of drawing a disc that can be multiplied by 6 is:

$P(6_m)=\frac{14}{85}=0{,}16$

d) There is only one element in this set and so the probability of drawing 65 is:

$P(65)=\frac{1}{85}=0{,}01$

e) The set of all discs that can be multiplied by 5 is: {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85}. This set contains 17 elements. Therefore the number of discs that cannot be multiplied by 5 is: $85-17=68$.

The probability of drawing a disc that cannot be multiplied by 5 is:

$P(\text{ not }5_m)=\frac{68}{85}=0{,}80$

f) In part b, we worked out the probability for a disc that is a multiple of 3. Now we work out the number of elements in the set of all discs that can be multiplied by 4: {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84}. This has 28 elements.

The probability that a disc can be multiplied by either 3 or 4 is:

$$\begin{aligned}P(3_m\cup 4_m)&=P(3_m)+P(4_m)-P(3_m\cap 4_m)\\&=0{,}33+\frac{28}{85}-0{,}33\times\frac{28}{85}\\&=0{,}55\end{aligned}$$

g) The set of all discs that are a multiple of 2 and 6 is the same as the set of all discs that are a multiple of 6. Therefore the probability of drawing a disc that is both a multiple of 2 and 6 is:

0,16

h) There is only 1 element in this set and so the probability is 0,01.

Random Experiments

1. AS, containing X and Y inside.

   Only X: 4,6,8,10,12,14,16
   Only Y: 3,5,7,11,13
   X and Y: 2
   In S but neither X nor Y: 1,9,15
   B
   n(S) = 16
   n(X) = 8
   n(Y) = 6
   n(X U Y) = 1
   n(X n Y) = 13

2. A - Three overlapping circles. Label them M, G and H respectively
   B - Each student must do exactly one of the following:
   - Take only geography;
   - Only take maths and/or history;

   There are 30 + 36 - 16 = 50 students doing the second one, therefore there must be 79 - 50 = 29 only doing geography.
   Each students must do exactly one of:

   - Only take geography;
   - Only take maths;
   - Take history;
   - Take geography and maths, but not history;

   There are 29, 8 and 36 of the first three. so the answer to B is:
   79 -29 - 8 - 36 = 6 people

   C: Calculated already: 29

   D: Each student must do exactly one of

   - Do geography
   - Only do maths
   - Only do history
   - Do maths and history but not geography

   Using the same method as before, the number of people in the last group is:
   79 - 41 - 8 - 16 = 14

3. A: S = {1,2,...,12}
   B: A = {1,2,3,4,6,12}
   C: B = {2,3,5,7,11}
   D: obvious
   E i)12
   ii)6
   iii)5
   iv)2
   v)9
   F yes

End of Chapter Exercises

1. 45 (All) - 6 (only Frosties) - 31 (both) = 8 (only Strawberry Pops)

2. A: 7/42 = 1/6

   B: Since 42 - 3 = 39 had at least one, and 23 + 7 had a packet of chips, then 39 - 30 = 9 only had Fanta.

   9/42 = 3/14

3. Draw venn diagram with two intersecting sets inside: one for multiples of 5, the other for odd numbers.

   Multiples of 5: 5

   Odd number: 1,3,5

   Neither: 2,4,6

   Both: 5

   A: 1/6

   B: 3/6 = 1/2

   C: 2/6 = 1/3

4. A: 1-7/12 = 5/12

   B: Error in question – no answer

5. A 190/300 = 19/30

   B 1 - 19/30 = 11/30

6. A: 8/18 = 4/9

   B: 1-4/9 = 5/9

   C: 2/18 = 1/9

   D: 1-1/9 = 8/9

   E: 1/9 + 4/9 = 5/9

   F: 1 - 5/9 = 4/9

7. A: Total number of biscuits is 9 + 4 +11 +18 = 42

   4/42 + 18/42 = 22/42 = 11/21

   B: 1 - 9/42 = 1 - 3/14 = 11/14

8. A: 15/280 = 3/56

   B: 1 - 3/56 = 53/56

9. A: All 4 groups are mutually exclusive, so total number of children is 44 + 14 + 5 + 40 = 103

   B i) 19/103

   ii) 58/103

   C 14/(14+5) = 14/19

10. A: i) Same as not black: 1 - 3/5 = 2/5

    ii) 3/5

    iii)1/7

    B: 1/7 * 70 = 10

    C: Typo in question?

11. A: Overlapping circles
B: Sample space divided into two.

12. Draw the picture.
Left side equals area covered by A and B, right side is A, plus B and removing double counted bit in the middle.

13. A {deck of card without clubs}
B P = {J,Q,K of hearts, diamonds or spades}
C: N = {A,2,3,4,5,6,7,8,9,10 or hearts, diamonds or spades}
D: (Mutually exclusive and complementary)
E: Complementary

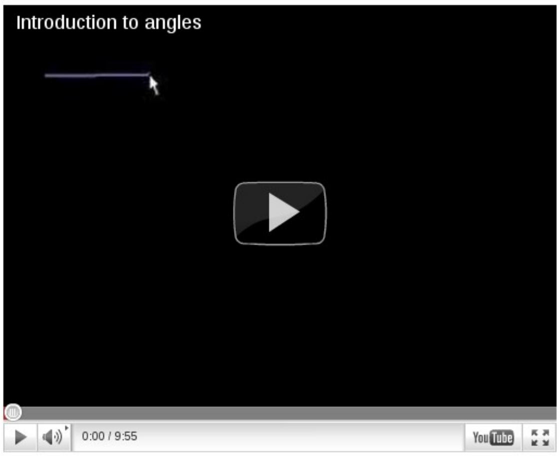
14. A: {Orange,Purple,Pink}
B: {Pink}
C: O = {Orange}, B = {Purple}
D: Three way split

Chapter 13 - Geometry Basics

http://cnx.org/content/m38380/latest/?collection=col11306/latest

Videos

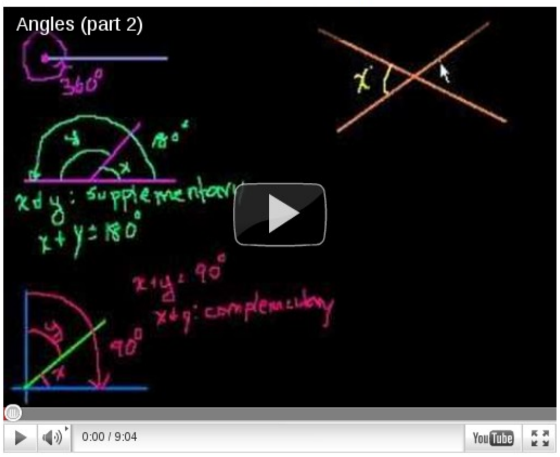
Khan Academy video on angles - 1



This Khan Academy video on Introduction to Angles covers the basic concept of an angle and then moves onto a few examples of angles, and introduces the concept of complementary and supplementary angles.

This video can be downloaded at: http://www.fhsst.org/lC4

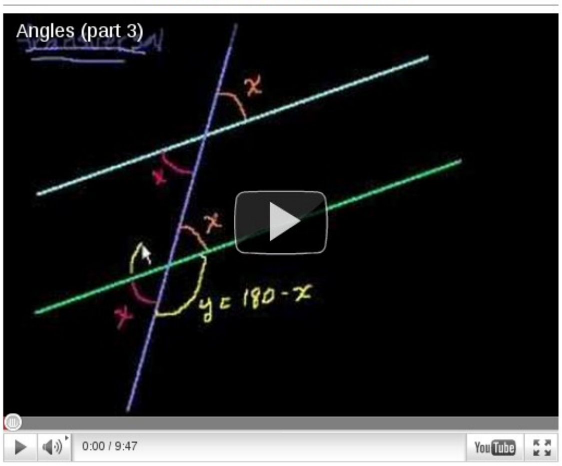
Khan Academy video on angles - 2



This Khan Academy video on Angles (part 2) reviews the concept of angles and supplementary and complementary angles. The video then goes on to explain angles around a point and to show that opposite angles are equal.

This video can be downloaded at: http://www.fhsst.org/lC2

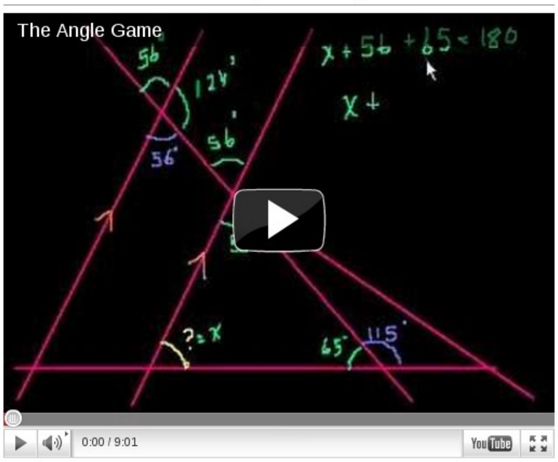
Khan Academy video on angles - 3



This Khan Academy video on Angles (part 3) covers some more concepts on angles. This video explains the concept of parallel lines and a transversal of the line. It then goes on to show how the various angles relate to each other and which angles are equal. Finally this video gives a very brief introduction to the angles in a triangle.

This video can be downloaded at:
http://www.fhsst.org/lCT

Khan Academy video on angles - 4



This Khan Academy video on the Angle Game covers two problems on finding the angles. This video shows how learners can take a complex problem and break it down into smaller sized pieces and slowly find all the required angles.

This video can be downloaded at:
http://www.fhsst.org/lCb

Solutions

Angles

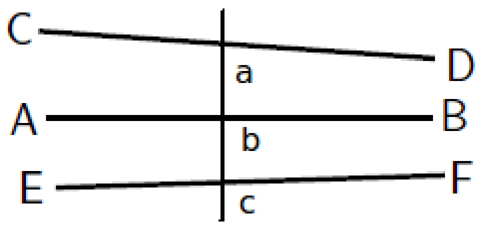
1. $a=180-30=150^\circ$ (adjacent angles on a straight line, linear pair)
   $b=30^\circ$ (opposite angles)
   $c=150^\circ$ (opposite angles)
   $d=180-30=150^\circ$ (d and b are co-interior angles)
   $e=180-150=30^\circ$ (d and e are adjacent angles on a straight line or e and b are adjacent angles.)
   $g=30^\circ$ (g and e are opposite angles)
   $f=150^\circ$ (f and d are opposite angles or c and f are corresponding angles)

2. (adjacent angles on a straight line)
$E\hat{F}B=180-150=30^\circ$ (co-interior angles)
$\hat{G}_3+\hat{G}_2=\hat{C}_2+\hat{C}_1$ (alternate interior angles) but $\hat{G}_2=\hat{C}_2$ so $\hat{G}_3=\hat{C}_1=100^\circ$ ($\hat{G}_3=C\hat{D}G$ (alternate interior angles))
$A\hat{B}F=180-30=150^\circ)-100=50^\circ$ (angles in a triangle)
$C\hat{F}G=180-50-30=100^\circ$ (angles on a straight line)
$F\hat{C}G=50^\circ$ (alternate interior angles)
$G\hat{C}D=180-50-100=30^\circ$ (angles on a straight line)
$F\hat{G}C=30^\circ$ (angles in a triangle)
$C\hat{G}D=50^\circ$ (angles in a triangle)
$D\hat{G}H=100^\circ$ (alternate interior angles)
$\hat{D}_1=80^\circ$ (angles on a straight line)

3. $x=\frac{20}{3}=6{,}667$

4. a) OP and QR are not parallel as the co-interior angles do not add up to 180. So there are no parallel lines in the figure.
b) MN and OP are not parallel. ($\hat{K}_2=35^\circ$ and so the corresponding angles are not equal)
c) $\hat{U}_1=85^\circ$ and so the corresponding angles are equal. This means that TY is parallel to MN ($TY \parallel MN$)

5. We construct a line that intersects all three lines:



$\hat{a}=\hat{b}$ since AB is parallel to CD (corresponding angles) and
$\hat{b}=\hat{c}$ since AB is parallel to EF (corresponding angles).
This means that $\hat{a}=\hat{c}$ and so CD is parallel to EF (corresponding angles)

Triangles

1. a) The triangle is isosceles and so x = y.

$$180=36-2x$$
$$2x=144$$
$$x=72^\circ=y$$

b) x is an exterior angle and so:

$$30+68=x$$
$$x=98^\circ$$

c) (angles on a straight line)
$$x+68=y$$
$$x=112-68$$
y is an exterior angle: $x=44^\circ$

d) Interior angles of a triangle add up to thus and . and (opposite angles) therefore $\hat{P}=\hat{S}$. The triangles Δ NPO and Δ ROS are thus similar because they have the same angles. Similar triangles have proportional sides thus $\frac{NP}{RS}=\frac{NO}{OR}$ i.e. $\frac{19}{76}=\frac{x}{116}$ so $x=19$ mm.

e) From the theorem of Pythagoras we get:

$$x^2=15^2+20^2$$
$$x=\sqrt{625}$$
$$x=25mm$$

f) Δ NPO and Δ RST are similar because they have the same angles, thus their sides are proportional and we can say i.e. so $x=18$ mm.

g) From the theorem of Pythagoras we get:

$$x^2=15^2-9^2$$
$$x=\sqrt{144}$$
$$x=12mm$$

And

$$y^2=x^2+5^2$$
$$y^2=144+25$$
$$y=\sqrt{169}$$
$$y=13mm$$

2. a) Congruent by SAS. (BC = CD, EC = AC and $B\hat{C}A=E\hat{C}D$)
   b) We have two equal sides ($AB=BD$, BC is shared) and one equal angle ($\hat{A}=\hat{D}$) but the sides do not include the known angle and therefore are not SAS and are not congruent . (Beware $A\hat{C}B$ is not necessarily equal to $D\hat{C}B$ since we are not given that $BC\perp AD$)
   c) There is not enough information given. We need at least three facts about the triangles and in this example we only know two sides in each triangle.
   d) There is not enough information given. Although we can work out which angles are equal we are not given any sides as equal. All we know is that we have two isosceles triangles. Note how this question differs from part A. In part A we were given equal sides in both triangles, in this question we are only given that sides in the same triangle are equal.
   e) Congruent by AAS. (CA is a common side, and two angles are given as being equal.)

Exercises

1. a) AB parallel to CD (alternate interior angles are equal.)
   b) NP is not parallel to MO (corresponding angles are not equal).

   MN is parallel to OP (corresponding angles are equal)

   c) GH parallel to KL (corresponding angles are equal)

   GK parallel to HL (corresponding angles are equal)

2. a) d = 73° (corresponding angles). $c=180-73=107^\circ$ (angles on straight line)
$a=180-73=107^\circ$ (co-interior angles) $b=180-107=73^\circ$ (co-interior angles)
b) a = 80° (angles on a straight line)
b = 80° (alternate interior angles)
c = 80° (corresponding angles)
d = 80° (opposite angles)
c) a = 50° (alternate interior angles)
b = 45° (alternate interior angles)
c = 95° (sum of interior angles)
d = 85° (sum of angles in a triangle.)

3. a) Congruent by SSS
b) Congruent by RHS

c) Congruent by AAS

d) Congruent by SAS

4. a) straight angle
b) obtuse angle

c) acute angle

d) right angle

e) reflex angle

5. a) $180-90-65=x=25^\circ$
b) $180-60-60=x=60^\circ$
c) $180-50-20=x=145^\circ$
d) $180-45-45=x=90^\circ$

6. a) circle; 1 side; not regular
b) decagon; 10 sides; regular

c) square; 4 sides, regular

d) hexagon; 6 sides; regular

7. a) True
b) False – the smallest angle that can be drawn is 0
c) False – an angle of 90 degrees is called a right angle
d) True
e) True
f) False – a regular polygon has equal angles and equal sides
g) False – an equilateral triangle has 3 equal sides
h) False – if 3 sides of a triangle are equal in length to the same sides of another triangle,

then the two triangles are congruent
l) True

8. a) acute angle.
b) acute angle.

c) right angle.

d) obtuse angle.

e) reflex angle.

f) revolution angle.

g) straight angle.

9.
a) $x=\sqrt{3^2+4^2}=\sqrt{9+16}=\sqrt{25}=5cm$

b) $x=\sqrt{2^2+7^2}=\sqrt{4+49}=\sqrt{53}\approx7.3cm$

c) $x=\sqrt{13^2-5^2}=\sqrt{139-25}=\sqrt{144}=12cm$

Challenge Problem

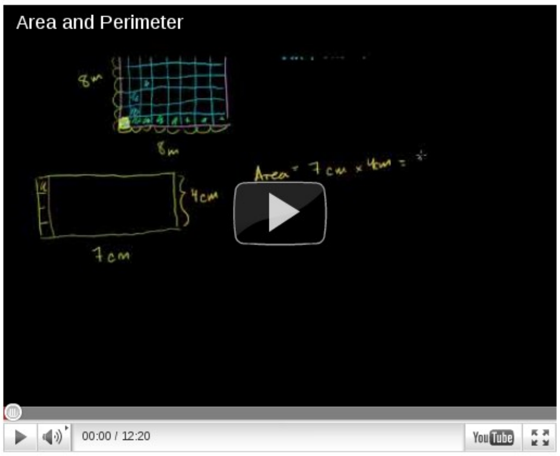
1. DE parallel to BC
e = c (alternate interior angles)
d = b (alternate interior angles)
We know that $d+a+e=180^\circ$
And we have shown that e = c and d = b, so we can replace d and e in the above to get:
$a+b+c=180^\circ$
So the angles in a triangle do add up to 180°

Chapter 14 - Geometry

http://cnx.org/content/m38381/latest/?collection=col11306/latest

Videos

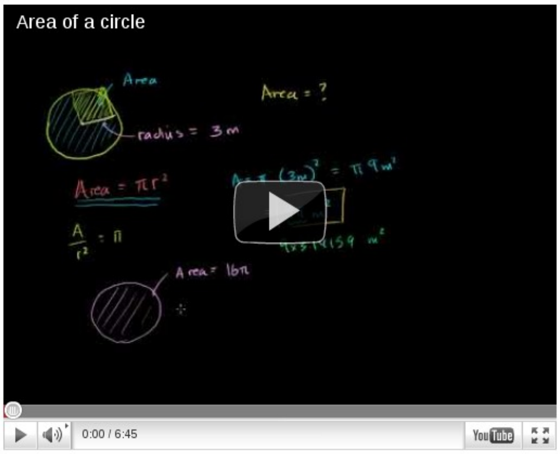
Khan Academy video on area and perimeter



This Khan Academy video on Area and Perimeter covers the concept of area and perimeter applied to triangles and rectangles. This video shows how learners can calculate the area and perimeter of rectangles and triangles.

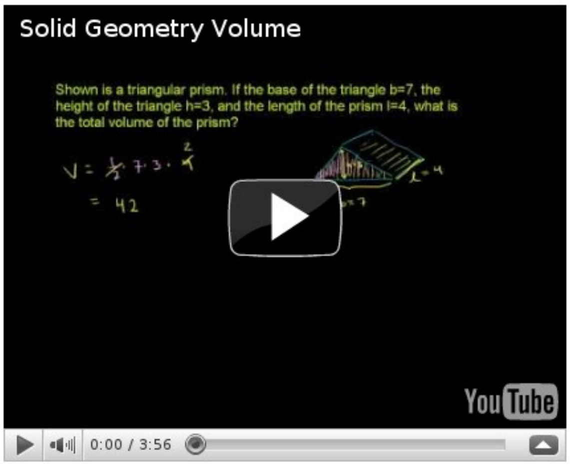
This video can be downloaded at: http://www.fhsst.org/lCj

Khan Academy video on area of a circle



This Khan Academy video on Area of a Circle covers the concept of the area of a circle. Several examples of working out the area of a circle are worked through in this video. The circumference of a circle is briefly touched on.

This video can be downloaded at: http://www.fhsst.org/lCD



This Khan Academy video on Solid Geometry Volume covers the concept of working out the volume of a solid. A sample problem is worked through in this video.

This video can be downloaded at: http://www.fhsst.org/lb1

Solutions

Polygons – Mixed

1. (Answers correlate to diagrams as read left to right, top to bottom)

ABCD a square (4 equal sides, 90º angle)

$$\therefore x=45^\circ$$
$$y=90^\circ$$
$$30\text{mm}=\sqrt{a^2+b^2}$$
$$=\sqrt{2a^2}$$
$$\therefore 2a^2=900\text{mm}$$
$$\therefore a^2=450\text{mm}$$
$$\therefore a=b=21\text{mm}$$

In EFGH sum of interior angles $=(\text{ number of sides }-2)\times 180^\circ$

$$=360^\circ$$
$$\therefore y=360^\circ-160^\circ-15^\circ-95^\circ$$
$$=90^\circ$$

in ILJK $c=180^\circ-65^\circ=115^\circ$ (parallel lines, comp angles)

$a=65^\circ$ (parallel lines, similar triangles)

$$b=180-a=115^\circ$$

Sum of angles $=(n-2)\times 180^\circ=3(180^\circ)=540^\circ$

$$\therefore 5x=540^\circ$$
$$\therefore x=108^\circ$$
$$y=180^\circ-x=180^\circ-108^\circ=72^\circ$$

In MNPO $\Delta\, M\hat{N}O \equiv \Delta\, O\hat{P}M$

(parallel lines $\therefore$ 3 equal sides)

$$\therefore b=50^\circ$$

Similarly $\Delta\, P\hat{M}N \equiv \Delta\, N\hat{O}P$

$$\therefore a=c$$

sum angles $=(n-2)180^\circ=360^\circ$

$$\therefore a+b+c+50^\circ=2a+100^\circ=360^\circ$$
$$\therefore 2a=260^\circ$$
$$\therefore a=c=130^\circ$$

In QRST $a=45^\circ$ (right-angled triangles)

$\sqrt{b^2+b^2}=20\text{mm}$ (Similar triangles, Pythagoras)

$$\therefore b=14.14\text{mm}$$

$b^2+2b^2=c^2$

c = 24,5 mm

In UVWZ sum of interior angles $=(n-2)\times 180^\circ$

$$=360^\circ$$
$$\therefore x=360^\circ-30^\circ-25^\circ-210^\circ$$
$$=95^\circ$$

In ABCD $a=45^\circ$ (sum of angles in isoceles triangle)

$b=\sqrt{25^2+25^2}$ (by Pythagoras)

$$=35.36\text{mm}$$

2. a)

$a=35$

$b=70$

b)

$$a=\sqrt{\left(\frac{WY}{2}\right)^2+\left(\frac{XZ}{2}\right)^2}$$

$$=\sqrt{65^2+30^2}$$

$$=71.6$$

c)

$a=120^\circ$

$b=10^\circ$

$c=d=50^\circ$

d)

$a=3$

e)

$a=15$

$b=100$

f)

$b-2=9$

$\therefore b=11$

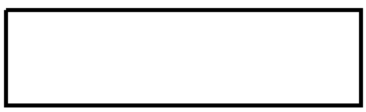
$a+3=6$

$\therefore a=3$

Polygons

1. a) True

b) False.



This is a rectangle that has length a and width b. $b \neq a$

c) True

d) True

e) False. You can draw pentagons with different sizes of sides.

f) False. You can draw any two equilateral triangles with different sizes of sides.

2. $A=\frac{1}{2}\text{base}\times\text{height}$

$A=\frac{1}{2}(10)(5)$

a) $A=25cm^2$

$A=\text{length}\times\text{breadth}$

$A=(10)(5)$

b) $A=50cm^2$

$A=\pi r^2$

$A=3,14159(5^2)$

$A=78,5398cm^2$

c) $A\approx 79cm^2$

(Note that the radius is half the diameter)

$A=\text{base}\times\text{perpendicularheight}$

$A=(10)\left(\sqrt{5^2-3^2}\right)(\text{Pythagoras})$

$A=(10)(4)$

d) $A=40$

$h^2=10^2+8^2$

$h=\sqrt{164}$

e) $h=12,81cm$

$A=\frac{1}{2}\text{base}\times\text{height}$

$A=\frac{1}{2}(12,81)(20)$

$A\approx 128cm^2$

f) We first need to construct the perpendicular height. If we do this such that we divide the base in half we get:

$h^2=3^2+5^2$

$h=\sqrt{34}$

$h=5,83cm$

$A=\frac{1}{2}\text{base}\times\text{height}$

$A=\frac{1}{2}(6)(5,83)$

$A=17,5cm^2$

g) Once again we construct the perpendicular height. If we do this such that we divide the base in half we get:

$h^2=10^2+5^2$

$h=\sqrt{125}$

$h=11,18cm$

$A=\frac{1}{2}\text{base}\times\text{height}$

$A=\frac{1}{2}(10)(11,18)$

$A=60cm^2$

h)

$h^2=15^2+9^2$

$h=\sqrt{306}$

$h=17,49cm$

$A=\text{base}\times\text{height}$

$A=(30)(17,49)$

$A=525cm^2$

Surface Areas

1.

$$S.A.=2[(L\times b)+(b\times h)+(L\times h)]$$
$$=2[(8\times 6)+(6\times 7)+(8\times 7)]$$

a) $=292cm^2$

b) $radius=\frac{1}{2}diameter=4cm$

$$S.A.=2\pi r^2+\pi rh$$
$$S.A.=2(\pi)(4^2)+(\pi)(4)(10)$$
$$S.A.=226{,}19cm^2$$

c) The height of the triangular face is:

$$10^2-4^2=\sqrt{84}=8{,}17cm$$
$$S.A.=2\left(\tfrac{1}{2}b\times h\right)+2(H\times S)+(H\times b)$$
$$S.A.=(8)(9{,}17)+2(10)(20)+(20)(8)$$
$$S.A.=633{,}36cm^2$$

2.

a)

$$\text{Surface Area}=(\text{ area of bottom of pool })+2\times(\text{ area of long sides })+2\times(\text{ area of short sides })$$
$$=(4m\times 3m)+2\times(4m\times 2.5m)+2\times(3m\times 2.5m)$$
$$=12m^2+20m^2+15m^2$$
$$=47m^2$$
$$\therefore \text{ the painter needs } \frac{47m^2}{2m^2} \text{ litres of paint}$$
$$=23.5 \text{ litres of paint (rounded up to the nearest litre)}$$

b)

$$\text{Surface Area}=(\text{ area of bottom of reservoir })+(\text{ area of inside of reservoir })$$
$$=(\pi r^2)+(\text{ Circumference of base}\times\text{height of reservoir })$$
$$=(\pi r^2)+(2pir\times 2.5m)$$
$$\text{but } r=\frac{4m}{2}=2m$$
$$\therefore \text{ Surface Area}=(2\pi(2m)^2)+(2\pi\times 2m\times 2.5m)$$
$$=56.5m^2$$
$$\therefore \text{ the painter needs } \frac{57m^2}{2m^2} \text{ litres of paint}$$
$$=29 \text{ litres of paint (rounded up to the nearest litre)}$$

Volume

1.

a) $V=L\times b\times h$

b) $V=\frac{1}{2}\times h\times b\times H$

c) $V=\pi r^2\times h$

2.

$$V=L\times b\times h$$
$$=6cm\times 7cm\times 10cm$$

a) $=420cm^3$

$$V=\frac{1}{2}\times h\times b\times H$$
$$=\frac{1}{2}\times 5cm\times 10cm\times 20cm$$

b) $=500cm^3$

$$V=\pi r^2\times h$$
$$=\pi(5cm)^2\times 10cm$$

c) $=785.4cm^3$

3. $$\text{Surface Area} = 2\{(L\times b) + (b\times h) + (L\times h)\}$$
$$= 2\{(a\times a) + (a\times a) + (a\times a)\}$$
$$= 2(3a^2) = 6a^2$$

$$\text{Volume} = L\times b\times h$$
$$= a\times a\times a$$
$$= a^3$$

Surface Areas and Volume:

1.

a) We use the formulae for volume and surface area of a sphere to solve the problem:

$$V_{sphere} = \frac{4}{3}\pi r^3$$
$$V_{sphere} = \frac{4}{3}\pi (4)^3$$
$$V_{sphere} = 268{,}08$$
$$S.A. = 4\pi r^2$$
$$S.A. = 4\pi (4^2)$$
$$S.A. = 201{,}06$$

b) A hemisphere is simply half a sphere, so we divide the volume by 2. For the surface area, we note that although we can divide the surface area of a sphere by 2, this does not take into account the surface area of the circle on top of the hemisphere. So we must divide the surface area of a sphere by 2 and add the surface area of a circle to this.

The radius is 3.

$$V_{hemisphere} = \frac{\frac{4}{3}\pi r^3}{2}$$
$$V_{hemisphere} = \frac{4\pi (3)^3}{6}$$
$$V_{hemisphere} = 56{,}55$$
$$S.A. = \frac{4\pi r^2}{2} + 2\pi r^2$$
$$S.A. = \frac{4\pi (3^2)}{2} + 2\pi (3^2)$$
$$S.A. = 113{,}1$$

c) We use the formulae for volume and surface area to solve the problem:

$$S.A. = \pi r^2 + \pi r\sqrt{r^2 + h^2}$$
$$S.A. = \pi (7^2) + \pi (7)\sqrt{7^2 + 14^2}$$
$$S.A. = 153{,}94 + 344{,}22$$
$$S.A. = 498{,}16$$
$$V_{cone} = \frac{1}{3}\pi r^2 h$$
$$V_{cone} = \frac{1}{3}\pi (7^2)(14)$$
$$V_{cone} = 718{,}38$$

d) We calculate the volume and surface area for the cone and the hemisphere separately. For the volumes, we can simply add the two volumes together. For the surface area we do not add the surface area of the circle into the surface area calculation for the hemisphere and we subtract the surface area of the circle from the surface area of the cone. Then we can just add the two volumes together.

Cone:

$$V_{cone} = \frac{1}{3}\pi r^2 h$$
$$V_{cone} = \frac{1}{3}\pi (3^2)(5)$$
$$V_{cone} = 47{,}12$$
$$S.A. = \pi r^2 + \pi r\sqrt{r^2 + h^2} - 2\pi r^2$$
$$S.A. = \pi (3^2) + \pi (3)\sqrt{3^2 + 5^2} - 2\pi 3^2$$
$$S.A. = 28{,}27 + 54{,}96 - 56{,}55$$
$$S.A. = 26{,}68$$

Hemisphere:

$V_{hemisphere}=\frac{\frac{4}{3}\pi r^3}{2}$

$V_{hemisphere}=\frac{4\pi(3)^3}{6}$

$V_{hemisphere}=56,55$

$S.A.=\frac{4\pi r^2}{2}$

$S.A.=\frac{4\pi(3^2)}{2}$

$S.A.=56,56$

Cone + hemisphere:

$V=47,12+56,55=103,67$

$S.A.=26,68+56,56=83,24$

e) We first need to find the perpendicular height. To do this, we note that we have a right angled triangle, which we have two sides for. second side is simply $\frac{24}{2}=12$).

So the perpendicular height is:

$13^2-12^2=h^2$

$h^2=25$

$h=5$

So now we can apply the formulae for prisms with square bases.

For surface area we simply add the surface area of each face together. So we have 4 triangles and 1 square:

$S.A.=4\left(\frac{1}{2}b\times h\right)+l^2$

$S.A.=2(24)(5)+24^2$

$S.A.=816$

The volume is:

$V=\frac{1}{3}a^2h$

$V=\frac{1}{3}(24^2)(5)$

$V=960$

2. We need to work out the surface area of the Earth. Assuming that the Earth is a sphere we get:

$S.A.=4\pi r^2$

$S.A.=4\pi(6378)^2$

$S.A.=511185932,5km^2=5,1\times10^8km^2$

So now if we work out 29% of this, we will find the area of land not covered by water:

$5,1\times10^8\times0,29=1,48\times10^8km^2$

Transformations

1. a) (2;1)

b) (5;-4)

c) (-6;0)

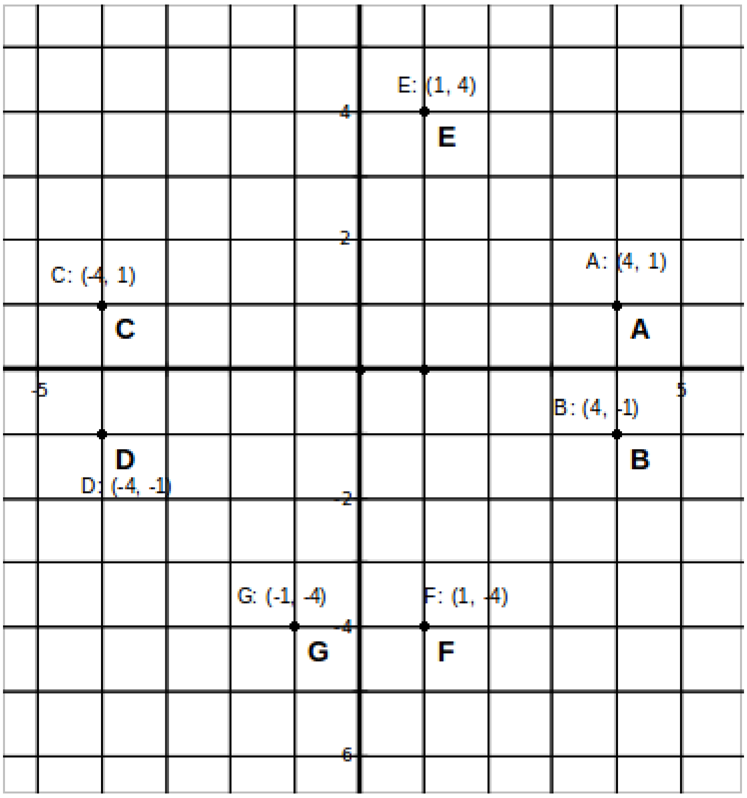
d) (3;0)

e) (0;3)

f) (-8;3)

g) (2;1)

2.



3. B: Reflection in x - axis; (-8;1)

C: Reflection in y – axis: (11; -4)

D: Reflection in y = -x ; (-2;-5)

End of Chapter Exercises

1. a) False. A trapezium has only one pair of parallel opposite sides. A parallelogram has two pairs of parallel opposite sides.

b) True.

c) False. A rectangle has all four corner angles equal to 90°.

d) False. The four sides of a rhombus have equal lengths.

e) True.

f) False. Two polygons are similar only if their corresponding angles are equal and if the ratios of corresponding sides are equal.

2. a) area $=1/2\times$ base $\times$ height $=1/2\times6\times3=9\text{cm}^2$

b) area $=$ base $\times$ height $=6\times5=30\text{cm}^2$

c) area $=$ base $\times$ height $=10\times10=100\text{cm}^2$

d) area $=1/2\times$(sum of parallel sides)$\times$ perpendicular height

$$\text{area}=1/2\times(10+13)\times4=46\text{cm}^2$$

e) area $=$ base $\times$ height $=10\times5=50\text{cm}^2$

f) area $=\pi\times\text{radius}^2=\pi\times4^2\approx50\text{cm}^2$

3.

a) $\text{S.A.}=2\pi r^2+2\pi rh=2\pi 4^2+2\pi(4)(10)\approx352\text{cm}^2$

$\text{Volume}=\pi r^2h=\pi(4^2)(10)\approx502\text{cm}^2$

b) $\text{S.A.}=2[(L\times b)+(b\times h)+(L\times h)]=2[(5\times4)+(4\times2)+(5\times2)]=72\text{cm}^2$

$\text{Volume}=L\times b\times h=5\times4\times2=40\text{cm}^3$

c) $\text{S.A.}=2\left(\frac{1}{2}b\times h\right)+2(H\times S)+(H\times b)=2\left(\frac{1}{2}(8)\times3\right)+2(20\times5)+(20\times8)=384\text{cm}^2$

$\text{Volume}=\frac{1}{2}\times h\times b\times H=\frac{1}{2}\times3\times8\times20=240\text{cm}^3$

4.

a) $\text{S.A.}=\pi r^2+2\pi r\sqrt{r^2+h^2}=\pi 3^2+2\pi(3)\sqrt{3^2+10^2}\approx225\text{cm}^2$

$\text{Volume}=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi(3^2)\times10\approx94\text{cm}^3$

b) $\text{S.A.}=4\left(\frac{1}{2}bh\right)+A=4\left(\frac{1}{2}\times15\times12\right)+15^2=585\text{cm}^2$

$\text{Volume}=\frac{1}{3}A\times h=\frac{1}{3}(15^2)\times12=900\text{cm}^3$

c) $\text{S.A.}=\frac{(4\pi r^2)}{2}\approx100\text{cm}^2$

$\text{Volume}=\frac{\left(\frac{4}{3}\pi r^3\right)}{2}=\frac{\left(\frac{4}{3}\pi 4^3\right)}{2}\approx134\text{cm}^3$

5. All the shapes will look the same after the transformation but their coordinates will be different

a)
$$\begin{aligned}A(x;y)&\to(2;2)\\B(x;y)&\to(6;0)\\C(x;y)&\to(2;-3)\\D(x;y)&\to(-1;0)\end{aligned}$$

b)
$$\begin{aligned}A(x;y)&\to(-2;5)\\B(x;y)&\to(0;2)\\C(x;y)&\to(-8;1)\end{aligned}$$

c)
$$\begin{aligned}A(x;y)&\to(-1;5)\\B(x;y)&\to(-3;6)\\C(x;y)&\to(-4;1)\end{aligned}$$

d) $$\begin{aligned}&A(x;y)\rightarrow(5;1)\\&B(x;y)\rightarrow(1;4)\\&C(x;y)\rightarrow(6;6)\end{aligned}$$

6.

a) $$\begin{aligned}&QR=\sqrt{(-2-3)^2+(5-2)^2}\\&=\sqrt{25+9}\\&=\sqrt{34}\end{aligned}$$

b) $M_{ps}=\frac{-3+2}{0-3}=\frac{1}{3}$

c) $Midpnt=(\frac{3}{2});(-\frac{1}{2})$

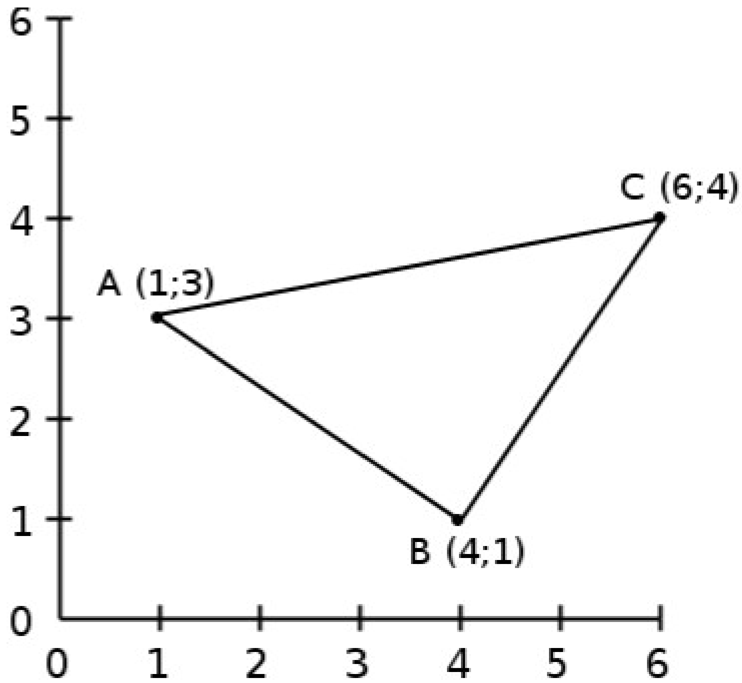
d) No, PQRS is not a parallelogram

7.

$$\frac{-2+2}{2}=a$$
$$a=0$$

$$\frac{3+6}{2}=b$$
$$b=\frac{9}{2}$$

8. a)



b) $d_{AB}=\sqrt{(4-1)^2+(1-3)^2}=\sqrt{13}$
$d_{BC}=\sqrt{(6-4)^2+(4-1)^2}=\sqrt{13}$
$d_{AC}=\sqrt{(6-1)^2+(4-3)^2}=\sqrt{26}$
Two sides are equal and so the triangle is isoceles.
c) $M_{AC}=(\frac{1+6}{2};\frac{3+4}{2})=(\frac{7}{2};\frac{7}{2})$
d) $m_{AB}=\frac{1-3}{4-1}=\frac{-2}{3}$
e) $m_{BD}=\frac{-1-1}{7-4}=\frac{-2}{3}$
$m_{AD}=\frac{-1-3}{7-1}=\frac{-2}{3}$
So A, B and D are collinear.

9\. a) $AB: y=\frac{1}{3}x+3$

b) $AB=\sqrt{40}$

c) A' = (7;-3) and B' = (-1;1)

d) $A'B': y=\frac{1}{3}x-\frac{2}{3}$

e) $A'B'=\sqrt{40}$

f) Yes, one pair of opposite sides is equal and parallel

10\. a) $P'(-1;2)$ $Q'(-1;8)$ $R'(2;4)$

11\.

a) True

b) False. A square and a rhombus are regular quadrilaterals but are not similar.

c) False. Pythagoras' theorem states that $AB^2+BC^2=CA^2$

d) True

e) False. Take a rectangle with length 3 and width 4 and compare it to a rectangle with length 6 and width 2. You should see that they are not similar.

12\.

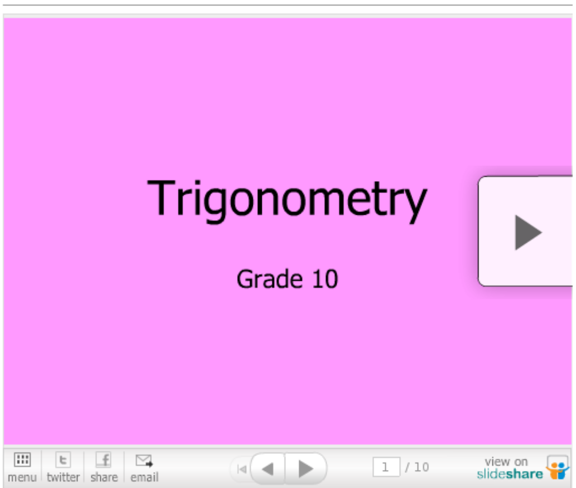
a) The triangles are similar as all angles in ABC have corresponding angles in PQR.

b) These rectangles are NOT similar because even though they have all the same angles their sides are not proportional.

Chapter 15 - Trigonometry

http://cnx.org/content/m38377/latest/?collection=col11306/latest

Presentations

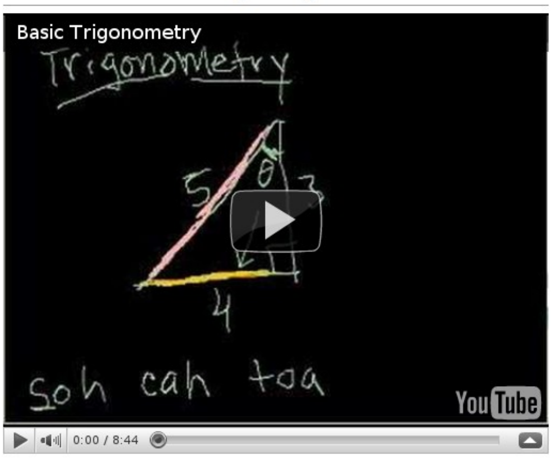


This presentation on Trigonometry covers the sine and cosine graphs, as well as the trigonometric ratios. A sample exercise on working out the sine, cosine and tangent of an angle is given.

This presentation can be downloaded and edited from: http://www.fhsst.org/lCK

Videos

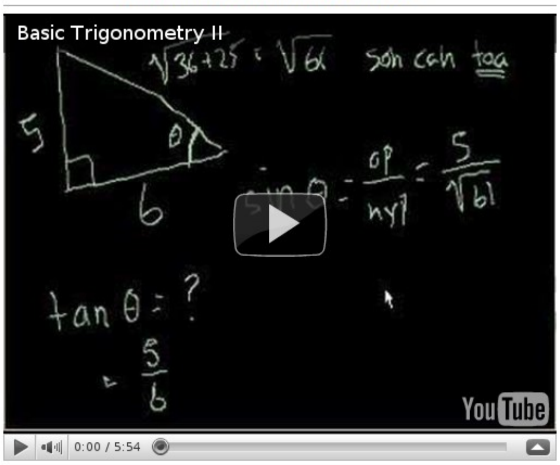
Trigonometry - 1



This Khan Academy video on Basic Trigonometry covers the trigonometric functions of sine, cosine and tangent and how to work these functions out for a right angled triangle. The mnemonic Soh Cah Toa is introduced as a way to remember the trigonometric functions.

This video can be downloaded from: http://www.fhsst.org/lCk

Khan academy video on trigonometry - 2



This Khan Academy video on Basic Trigonometry 2 covers several examples on working out the sine, cosine and tangent of an angle in right angled triangles.

This video can be downloaded from: http://www.fhsst.org/lC0

Solutions

Finding Lengths

1. $\sin(37)=\frac{a}{62}$

   $a=62\times\sin(37)$

   a) $a=37.31\text{units}$

   $\tan(23)=\frac{b}{21}$

   $b=21\times\tan(23)$

   b) $b=8.91\text{units}$

   $\cos(55)=\frac{c}{19}$

   $c=19\times\cos(55)$

   c) $c=10.90\text{units}$

   $\cos(49)=\frac{d}{33}$

   $d=33\times\cos(49)$

   d) $d=21.65\text{units}$

   $\sin(17)=\frac{12}{e}$

   $e=\frac{12}{\sin(17)}$

   e) $e=41.04\text{units}$

   $\cos(22)=\frac{31}{f}$

   $f=\frac{31}{\cos(22)}$

   f) $f=33.43\text{units}$

   $\cos(23)=\frac{g}{32}$

   $g=32\times\cos23$

   g) $g=29.46\text{units}$

   $\sin(30)=\frac{h}{20}$

   $h=20\times\sin30$

   h) $h=10.00\text{units}$

Applications of Trigonometric Functions

1. $\cos(x)=\frac{30\text{m}}{50\text{m}}$

   $x=53.13^{\circ}$

   The angle of elevation is $53{,}13^{\circ}$

2. $\tan(x)=\frac{7.15\text{m}}{10.1\text{m}}$

   $x=35.30^{\circ}$

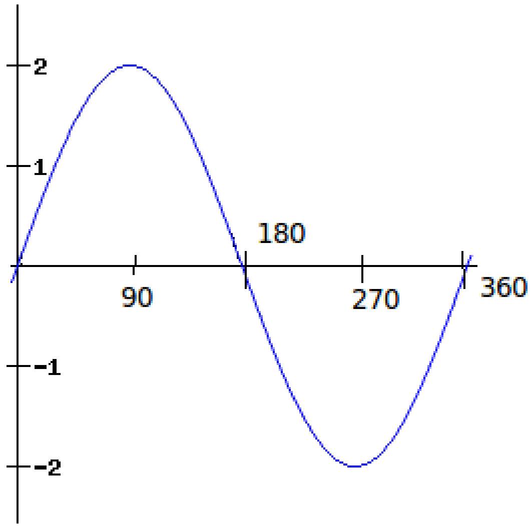
   The angle of elevation of the sun is: $35{,}30^{\circ}$

Graphs of Trigonometric Functions

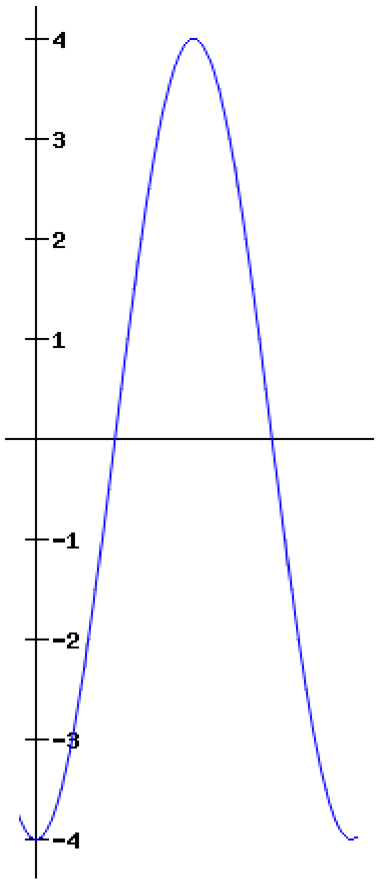
1. a) The 2 in front of the cos stretches the graph by 2 units in the y direction.

   b) The -4 in front of the cos stretches the graph by 4 units in the y direction and inverts (turns upside down) the graph.

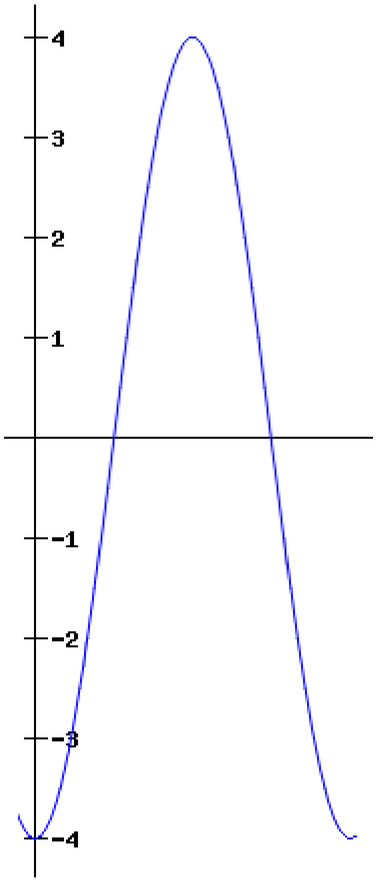
a)

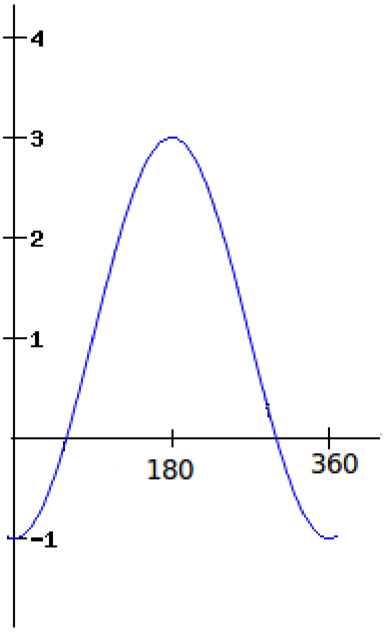


b)

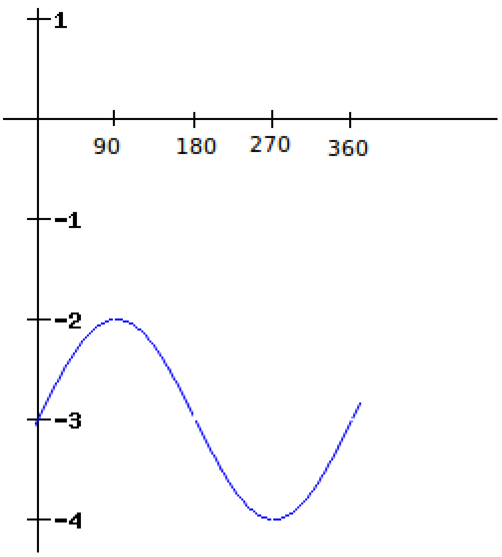


c) The -2 has a similar effect to the -4 in the previous graph, the +1 shifts the graph 1 unit up on the y-axis.

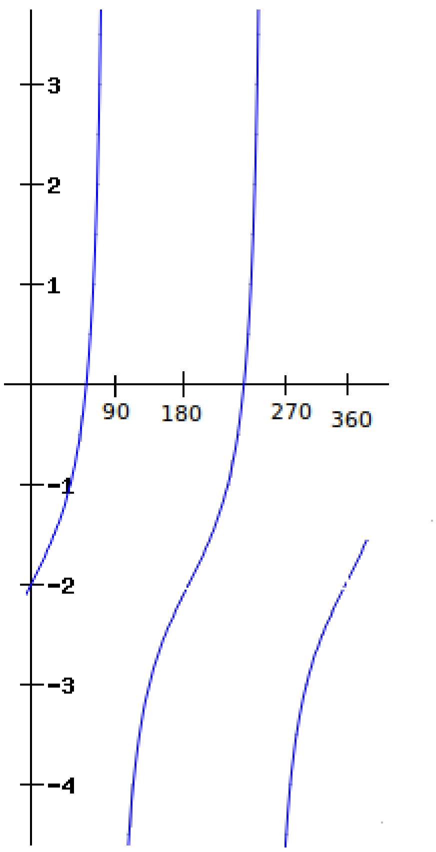




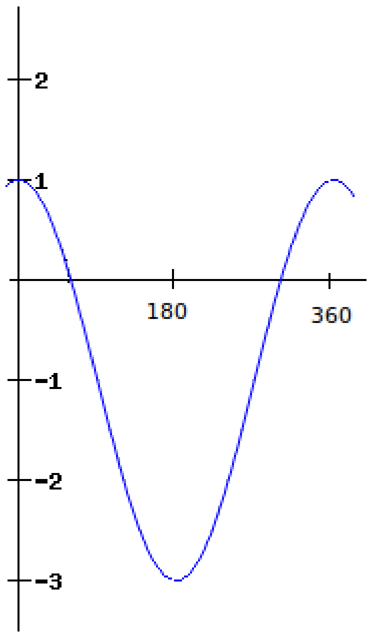
d) The -3 shifts the sine graph 3 units down on the y-axis.



e) The -2 shifts the tan graph 2 units down on the y-axis.



f) The 2 in front of the cos stretches the graph by 2 units on the y-axis and the -1 shifts the graph 1 unit down on the y-axis.



2.

a) $y=2\cos(\theta)$

From the shape of the graph and the intercepts we determine that this is a cos graph. From the y-intercepts we see that it has been stretched by a factor of 2.

b) $y=\sin(\theta)+1$

From the shape of the graph and the intercepts we determine that this is a sin graph. From the fact that the graph lies above the x-axis we see that q must be +1

c) $y=-\tan(\theta)+5$

From the shape of the graph we determine that this is a tan graph. From the y-intercept we see that it has been shifted up by 5 units.

End of Chapter Exercises

1. For all of these we use the appropriate trig function. Also note that we sometimes use the theorem of Pythagoras to find the third side.

   To find a and b we use $\cos(\theta)=\frac{adj}{hyp}$

   $\cos(30)=\frac{a}{16}$

   $a=16\cos(30)$ $\cos(25)=\frac{b}{13,86}$

   $a=13,86cm$ $b=12,56cm$

   To find c we use $\sin(\theta)=\frac{opp}{hyp}$

   $\sin(20)=\frac{c}{12,56}$

   $c=4,30cm$

   To find d we use $\cos(\theta)=\frac{adj}{hyp}$

   $\cos(50)=\frac{5}{d}$

   $d=7,779cm$

   Next we use the theorem of Pythagoras to find the third side, so we can use trig functions to find e.

   $5^2+7,779^2=85,51$

   $\sqrt{85,51}=9,247$

   We use $\tan(\theta)=\frac{opp}{adj}$ to find e.

   $\tan(60)=\frac{9,247}{e}$

   $e=5,339cm$

   Next we use the theorem of Pythagoras to find the third side, so we can use trig functions to find f and g.

   $5,339^2+7,779^2=89,018$

   $\sqrt{89,018}=9,435cm$

   We find g using $\tan(\theta)=\frac{opp}{adj}$

   $\tan(80)=\frac{9,345}{g}$

   $g=1,648cm$

   And finally we find f using the theorem of Pythagoras.

   $f^2=9,345^2-1,648^2$

   $f=\sqrt{84,613}$

   $f=9,2cm$

2. a) Since we are told that $PX\perp QR$ we can use $\cos(\theta)=\frac{adj}{hyp}$ to find XR.

   $\cos(30)=\frac{XR}{20}$

   $XR=20\cos(30)$

   $XR=17,32cm$

   b) We use $\sin(\theta)=\frac{opp}{hyp}$

   $\sin(30)=\frac{PX}{20}$

   $PX=20\sin(30)$

   $PX=10cm$

   c) We know the length of QR and we have found the length of XR, so we can work out the length of QX:

   $QX=QR-XR$

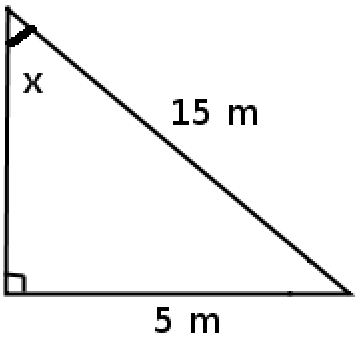
   $QX=4,68cm$

   Since we know two sides and an angle we can use $\tan(\theta)=\frac{opp}{adj}$ to find the angle.

   $\tan(Q\hat{P}X)=\frac{4,68}{10}$

   $Q\hat{P}X=25^{\circ}$

3. We first draw a diagram to help us understand the problem:



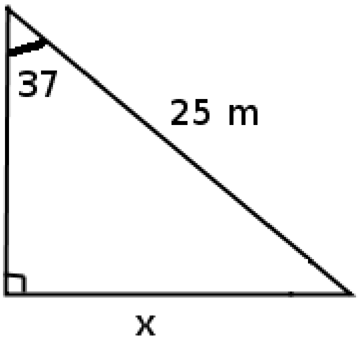
Notice that we want to find the angle that the ladder makes with the wall, not the angle that the ladder makes with the ground.

Now we use $\sin(x)=\frac{opp}{hyp}$

$\sin(x)=\frac{5}{15}$

$x=19{,}47^{\circ}$

4. We first draw a diagram to help us understand the problem:



Notice that we are given the angle that the ladder makes with the wall, not the angle that the ladder makes with the ground.

Now we can use $\sin(\theta)=\frac{opp}{hyp}$ to find x:

$\sin(37)=\frac{x}{25}$

$x=25\sin(37)$

$x=15m$

5. We use $\tan(\theta)=\frac{opp}{adj}$ to find DC:

$\tan(41)=\frac{9}{DC}$

$DC=9\tan(41)$

$DC=7{,}82units$

Next we find BC:

$BC=BD-DC$

$BC=9{,}18units$

And then we use $\tan(\theta)=\frac{opp}{adj}$ to find the angle:

$\tan(A\hat{B}C)=\frac{9}{9{,}18}$

$A\hat{B}C=44{,}43^{\circ}$

6. We use the angles in a triangle to find:

$C\hat{A}B=180-90-35=55^{\circ}$

Then we find $D\hat{A}B$:

$D\hat{A}B=15+55=70^{\circ}$

Now we can use $\tan(\theta)=\frac{\text{opp}}{\text{adj}}$ to find BC:

$\tan(35)=\frac{9}{BC}$

$BC=12,85\text{units}$

And then we find BD also using $\tan(\theta)=\frac{\text{opp}}{\text{adj}}$

$\tan(70)=\frac{BD}{9}$

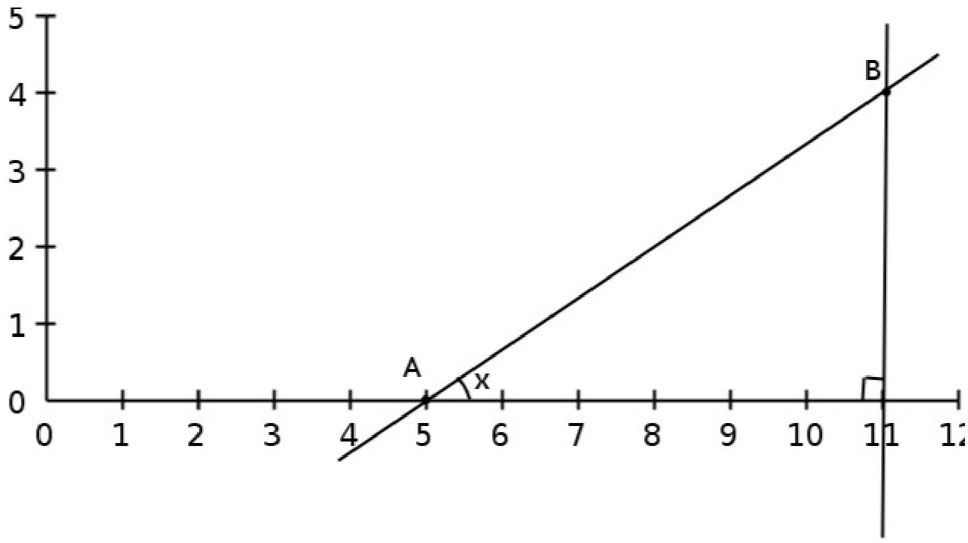
$BD=24,73\text{units}$

Finally we can find CD:

$CD=BD-BC$

$CD=11,88\text{units}$

7. We draw a diagram to help us understand the problem:



Next we note that the distance from B to the x-axis is 4 (B is 4 units up from the x-axis) and that the distance from A to C is 11 – 5 = 6.

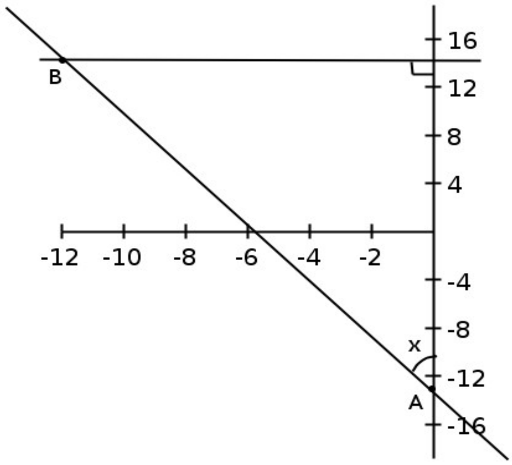
We use $\tan(x)=\frac{\text{opp}}{\text{adj}}$

$\tan(x)=\frac{4}{6}$

$x=33,69^{\circ}$

8.

We draw a diagram to help us understand the problem:



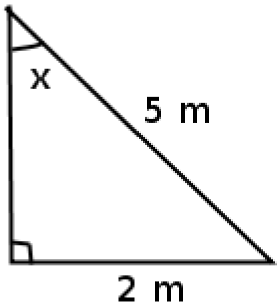
The distance from B to the y-axis is 12 units (although B is (-12;-14) the distance is positive). The distance from A to the point where the perpendicular line from B intercepts the y-axis is $14-(-13)=27$

Now we use $\tan(x)=\frac{\text{opp}}{\text{adj}}$

$\tan(x)=\frac{12}{27}$

$x=23,96^{\circ}$

9. We draw a diagram to help us understand the problem:



Notice that we want the angle that the ladder makes with the wall, not the angle that the ladder makes with the ground.

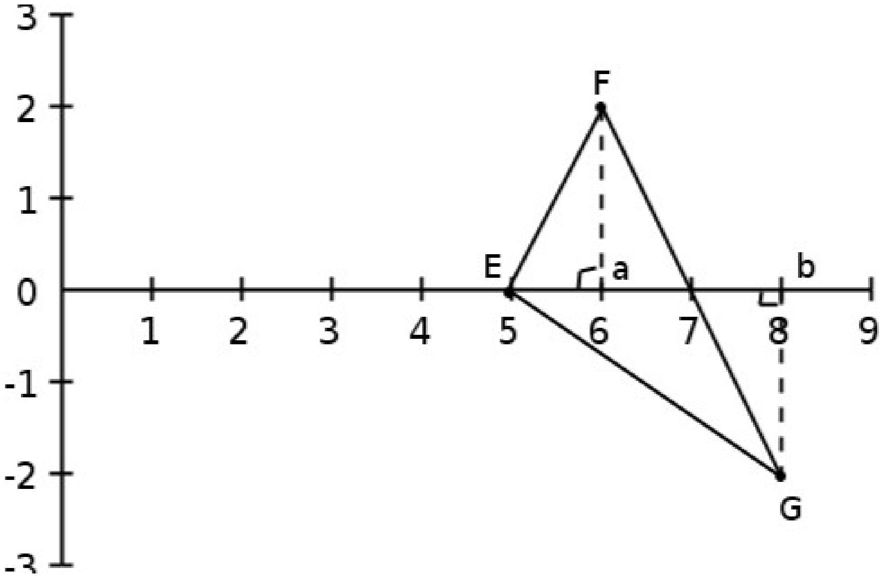
We use $\sin(x)=\frac{\text{opp}}{\text{hyp}}$

$\sin(x)=\frac{2}{5}$

$x=23,58^{\circ}$

10.

We draw a diagram to help us understand the problem:



To find $F\hat{E}G$ we look at triangles FEa and GEb in turn. These two triangles will each give one part of the angle that we want.

In triangle FEa we can use $\tan(\theta)=\frac{\text{opp}}{\text{adj}}$. Fa is 2 units and Ea is 1 unit. (F is 2 units up from the x-axis and using only the x co-ordinates, EF is 1 unit.)

$$\tan(F\hat{E}x)=\frac{2}{1}$$

$$F\hat{E}x=63{,}43^{\circ}$$

In triangle GEb we also use $\tan(\theta)=\frac{\text{opp}}{\text{adj}}$. Gb is 2 units and Eb is 3 units. (G is 2 units down from the x-axis and using only the x co-ordinates Eb is 3 units.)

$$\tan(G\hat{E}x)=\frac{2}{3}$$

$$G\hat{E}x=33{,}69^{\circ}$$

Now we add these two angles together to get the angle we want to find:

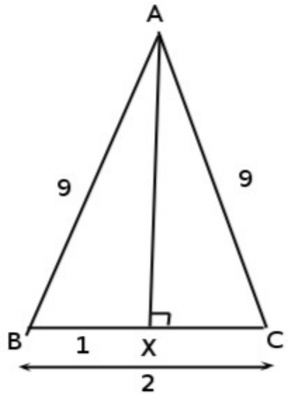
$$G\hat{E}x+F\hat{E}x=F\hat{E}G$$

$$F\hat{E}G=33{,}69+63{,}43$$

$$F\hat{E}G=97{,}12^{\circ}$$

We first draw a diagram to help us understand the problem:

11.



We construct a perpendicular bisector so that we have a right-angled triangle to work with.

In triangle ABX we find the angles using trig functions.

$$\cos(B\hat{A}X)=\frac{1}{9}$$

$$B\hat{A}X=6{,}4^{\circ}$$

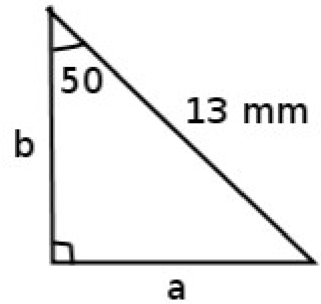
Since AX bisects $B\hat{A}C$ we find that $B\hat{A}C=2(6{,}4)=12{,}8^{\circ}$.

Since the triangle is an isosceles triangle the other two angles are equal. We can check that we have indeed found the smallest angle by calculating the remaining angles:

$$180-12{,}8=2A\hat{B}C$$

$$A\hat{B}C=83{,}6^{\circ}=A\hat{C}B$$

12. We draw a diagram to help us understand the problem:



Next we get:

$\sin(\theta)=\frac{\text{opposite}}{\text{hypotenuse}}$

$\sin(50)=\frac{a}{13}$

$a=13\sin(50)$

$a=9{,}96\text{mm}$

$b^2=c^2-a^2$

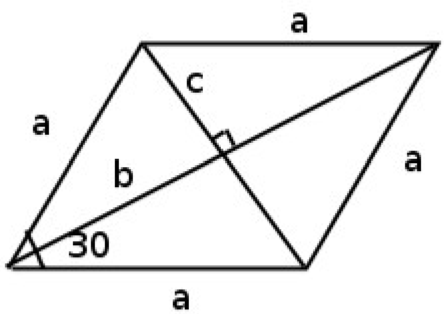
N… e theorem of Pythagoras to find the other side:

$b^2=13^2-9{,}96^2$

$b^2=69{,}7984$

$b=8{,}35\text{mm}$

13. First we draw the rhombus with its diagonals:



a) The perimeter is found by adding each side together. Since the sides are all equal this is:

$P=4a$

$20=4a$

$a=5\text{cm}$

b) The diagonals of a rhombus bisect the angle, so working in one of the small triangles we use the trig functions to find b:

$\cos(\theta)=\frac{\text{adjacent}}{\text{hypotenuse}}$

$\cos(15)=\frac{b}{5}$

$b=4{,}8\text{cm}$

Next we find the third side of the triangle:

$c^2=a^2-b^2$

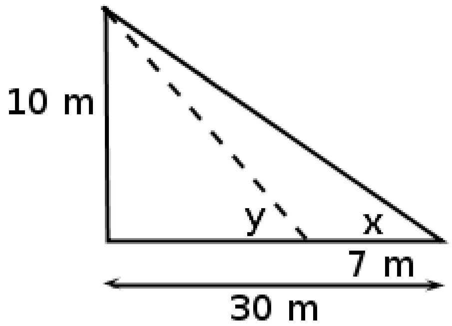
$c^2=25-23{,}33$

$c=1{,}29\text{cm}$

Since the diagonals bisect each other, we know that the total length of each diagonal is either 2b or 2c, depending which diagonal we are looking at.

The one diagonal is $4{,}8\times2=9{,}6\text{cm}$ and the other diagonal is $1{,}29\times2=2{,}58\text{cm}$.

14. We first draw a diagram to help us understand the problem:



The dashed line is for part b, when he has sailed 7 m towards the lighthouse.

$\tan(x)=\frac{opp}{adj}$

$\tan(x)=\frac{10}{30}$

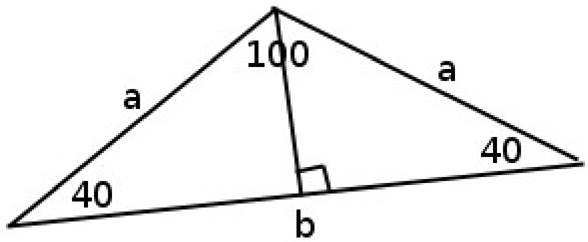
a) $x=18^{\circ}$

$\tan(y)=\frac{opp}{adj}$

$\tan(y)=\frac{10}{23}$

b) $y=23^{\circ}$

15. We draw a diagram to help us understand the problem:



We construct a perpendicular bisector and now have a right-angled triangle to work with. We can use either of these two triangles.

We know:

$2a+b=20$

Rearranging gives:

$b=2(10-a)$

We can use:

$\cos(\theta)=\frac{adj}{hyp}$

This gives:

$$\cos(50)=\frac{\frac{b}{2}}{a}$$
$$0{,}77=\frac{\frac{2(10-a)}{2}}{a}$$
$$0{,}77=\frac{10-a}{a}$$
$$0{,}77a=10-a$$
$$a=5{,}65\text{cm}$$

From the perimeter we get:

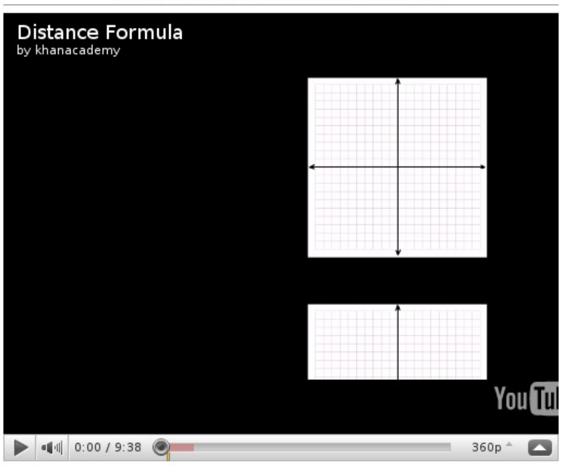
$b=2(10-5{,}65)=8{,}7\text{cm}$

Chapter 16 - Analytical Geometry

http://cnx.org/content/m38370/latest/?collection=col11306/latest

Videos

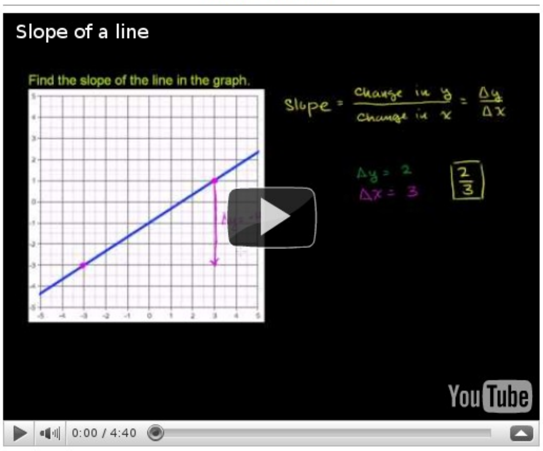
Khan academy video on distance formula



This Khan Academy video on the Distance Formula covers several examples of working out the distance between two points. This video also explains how to derive the distance formula and gives an easy way of remembering how to find the distance between two points.

This video can be downloaded at: http://www.fhsst.org/lCW

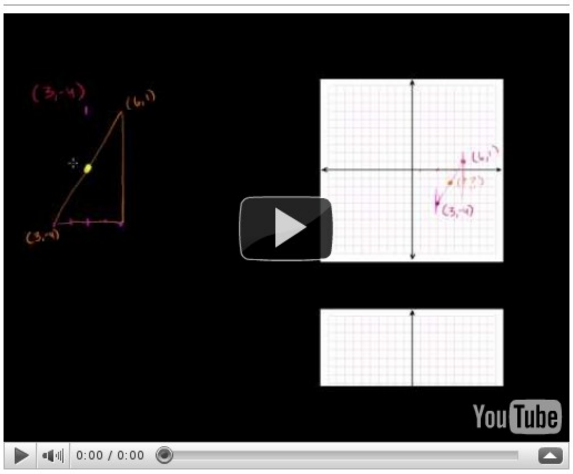
Gradient of a line



This Khan Academy video on Slope of a Line covers an example of finding the slope of a line. This video gives a formula for finding the slope of a line as well as an explanation of how to find the slope of a line.

This video can be downloaded at: http://www.fhsst.org/lCB

Khan academy video on midpoint of a line



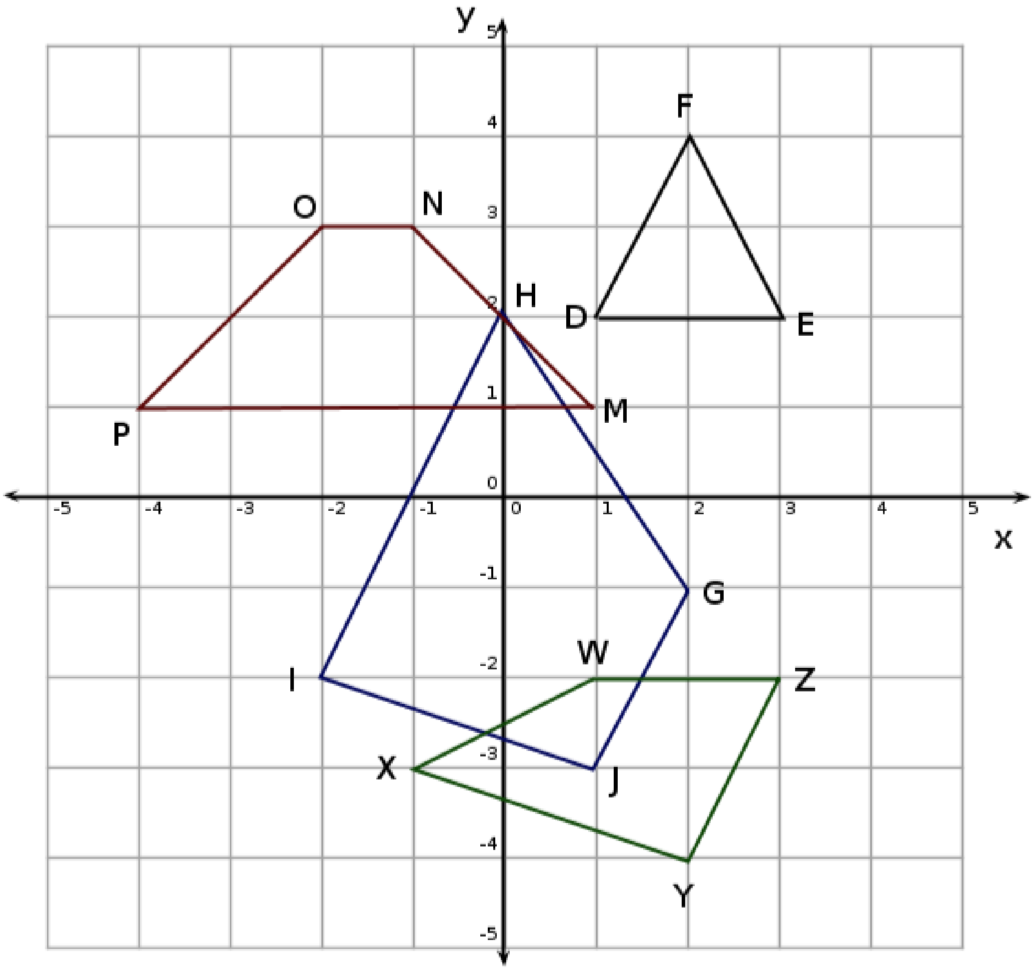
This Khan Academy video on Midpoint of a Line covers several examples on finding the midpoint of a line. This video gives a simple derivation of the formula for the midpoint of a line and an easy way to remember this formula.

This video can be downloaded at: http://www.fhsst.org/lCZ

Solutions

End of Chapter Exercises:

1a) The figures are given below.



2 a)

$$FG = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(1-2)^2 + (5-0)^2}$$
$$= \sqrt{(-1)^2 + (5)^2}$$
$$= \sqrt{26} = 5.1$$

b) No.

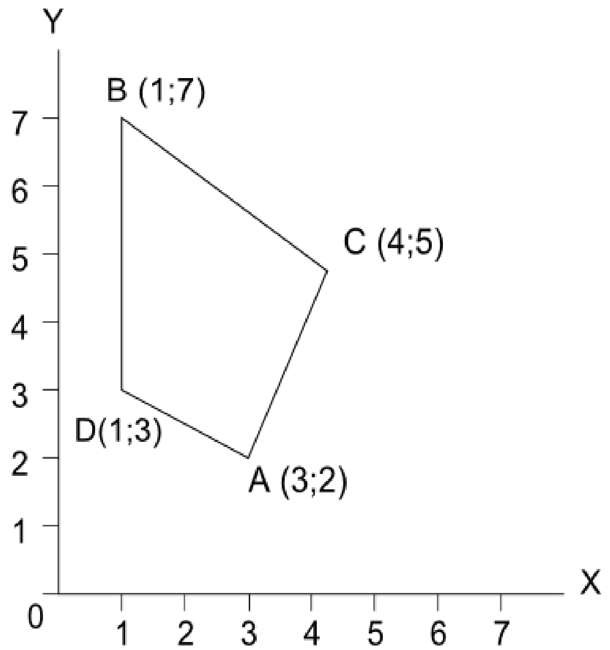
c)Midpoints not equal, therefore diagonals do not bisect.

$$\text{Midpoint FH} = \frac{(x_1 + x_2)}{2}; \frac{(y_1 + y_2)}{2}$$
$$= \frac{5}{2}; \frac{7}{2}$$
$$\text{Midpoint GI} = \frac{8}{2}; \frac{7}{2}$$

d) No. It has no parallel or equal sides, and no diagonal bisectors therefore it is just an ordinary quadrilateral.

3)

a)



b)

$$AD=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$
$$=\sqrt{(1-3)^2+(3-2)^2}$$
$$=\sqrt{(-2)^2+(1)^2}$$
$$=\sqrt{5}=2.24$$

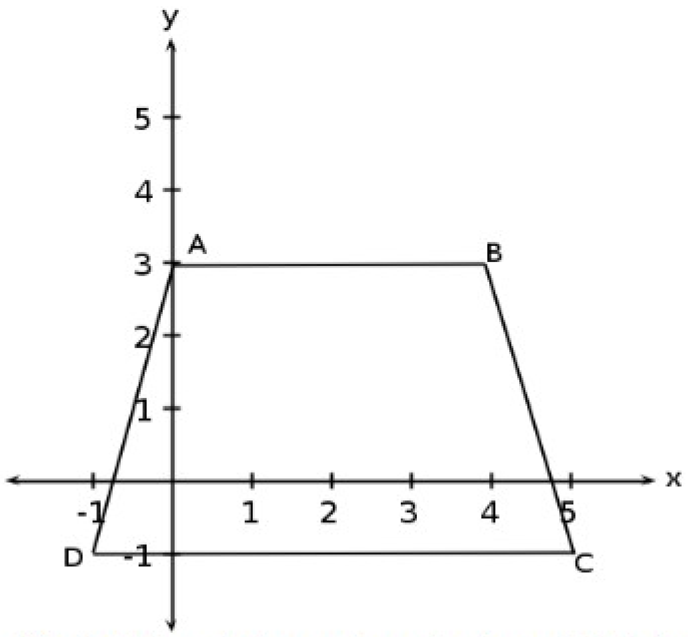
Similarly DB=4

BC=3.61

CA=3.16

4)

We first draw the figure:



ai) To show that these two lines are the same length we calculate the length of each line:

length AD:

$$\text{Distance}=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$
$$=\sqrt{(0-(-1))^2+(3-(-1))^2}$$
$$=\sqrt{1+16}$$
$$=\sqrt{17}$$

These two distances are the same and so: $AD=BC$

ii) For two lines to be parallel the gradients will be the same. So we find the gradients of the two lines and compare them:

gradient AB:

$$\begin{aligned}\text{gradient} &= \frac{y_2-y_1}{x_2-x_1}\\ &= \frac{3-3}{0-4}\\ &= \frac{0}{-4}\\ &= 0\end{aligned}$$

gradient DC:

$$\begin{aligned}\text{gradient} &= \frac{y_2-y_1}{x_2-x_1}\\ &= \frac{-1-(-1)}{5-(-1)}\\ &= \frac{0}{6}\\ &= 0\end{aligned}$$

Since the two gradients are the same the lines are parallel or: $AB \parallel DC$

b) Isosceles trapezium. It has one opposite pair of lines equal and one opposite pair of lines parallel.

c) To do this we find the midpoint of AC and BD. If these two results are not the same then the diagonals do not bisect each other.

Midpoint AC:

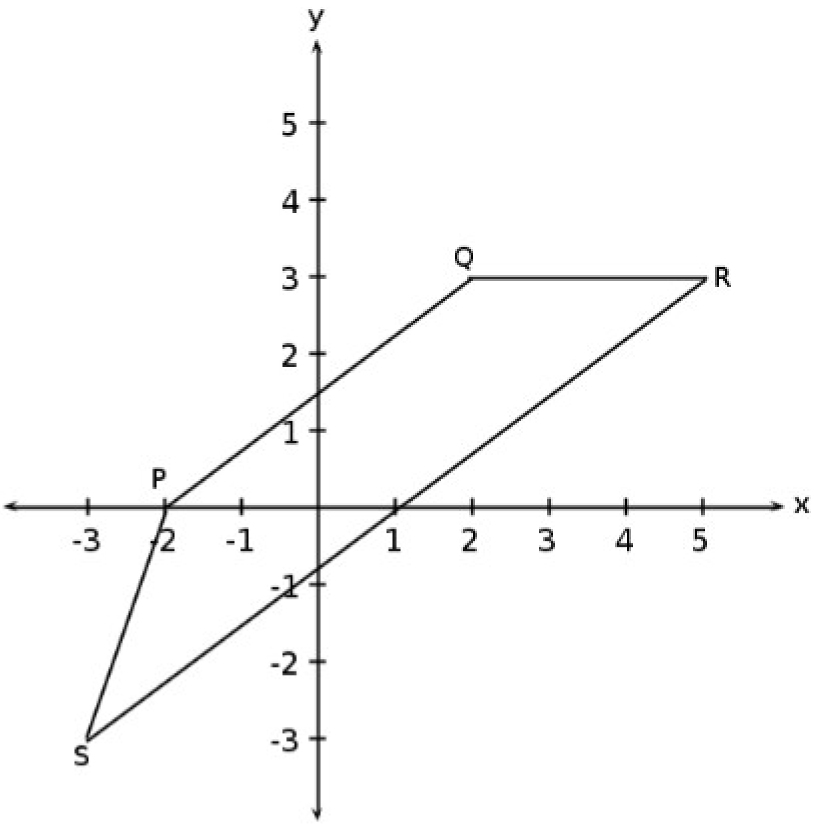
$$\begin{aligned}\text{midpoint} &= \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)\\ &= \left(\frac{0+5}{2};\frac{3+(-1)}{2}\right)\\ &= \left(\frac{5}{2};1\right)\end{aligned}$$

Midpoint BD:

$$\begin{aligned}\text{midpoint} &= \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)\\ &= \left(\frac{4+(-1)}{2};\frac{3+(-1)}{2}\right)\\ &= \left(\frac{3}{2};1\right)\end{aligned}$$

5)

We first draw the quadrilateral:



ai) We find the lengths of the two lines:

length PQ:

$$\text{distance} = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$$
$$= \sqrt{(-2-2)^2 + (0-3)^2}$$
$$= \sqrt{25}$$
$$= 5$$

length RS:

$$\text{distance} = \sqrt{(x_R - x_S)^2 + (y_R - y_S)^2}$$
$$= \sqrt{(5-(-3))^2 + (3-(-3))^2}$$
$$= \sqrt{100}$$
$$= 10$$

So RS = 2PQ

ii) We find the gradient of SR and the gradient of PQ. If they are the same then the lines are parallel.

gradient SR:

$$\text{gradient} = \frac{y_R - y_S}{x_R - x_S}$$
$$= \frac{3-(-3)}{5-(-3)}$$
$$= \frac{6}{8} = \frac{3}{4}$$

gradient PQ:

$$\text{gradient} = \frac{y_P - y_Q}{x_P - x_Q}$$
$$= \frac{0-3}{-2-2}$$
$$= \frac{-3}{-4} = \frac{3}{4}$$

So $PQ \parallel SR$

$$PS = \sqrt{(x_P - x_S)^2 + (y_P - y_S)^2}$$
$$= \sqrt{(-2-(-3))^2 + (0-(-3))^2}$$

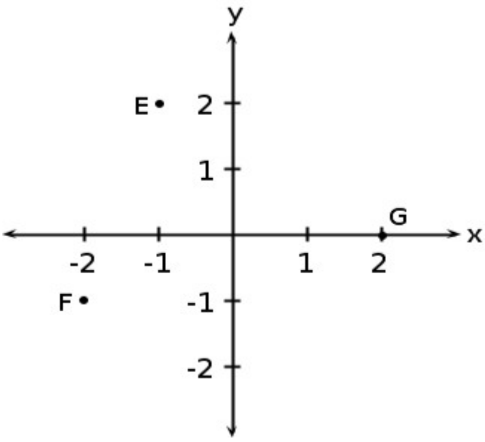
bi) $= \sqrt{10}$

ii)

$$QR = \sqrt{(x_Q - x_R)^2 + (y_Q - y_R)^2}$$
$$= \sqrt{(2-5)^2 + (3-3)^2}$$
$$= \sqrt{9}$$
$$= 3$$

c) Trapezium. One pair of opposite sides is parllel.

6) We plot the points given:



We now find the midpoint of EG. This will be the same as the midpoint of FH since the diagonals bisect each other.

Midpoint EG:

$$\text{midpoint} = \left(\frac{x_E + x_G}{2}; \frac{y_E + y_G}{2}\right)$$
$$= \left(\frac{-1+2}{2}; \frac{2+0}{2}\right)$$
$$= \left(\frac{1}{2}; 1\right)$$

Midpoint of FH:

x co-ordinate:

$$\frac{1}{2} = \frac{x_F + x_H}{2}$$
$$\frac{1}{2} = \frac{-2 + x_H}{2}$$
$$x_H = 2$$

y co-ordinate:

$$1 = \frac{y_F + y_H}{2}$$
$$2 = -1 + y_H$$
$$y_H = 3$$

So we have H(2;3)

Take note of the convention we followed for labelling a quadrilateral. We called the quadrilateral EFGH so EF, FG, GH and EH are the lines that define the quadrilateral. So, although there are three ways to draw a parallelogram from three points, two of these parallelograms would not be called parallelogram EFGH. They would be EGFH and EGHF. You can try see if you can find the co-ordinates of H that correspond to these parallelograms.

(The co-ordinates are: (-5;1) and (1;-3) respectively.)

7) a)
$$QR = \sqrt{(-2-3)^2 + (5-2)^2}$$
$$= \sqrt{25+9}$$
$$= \sqrt{34}$$

b) $M_{ps} = \dfrac{-3+2}{0-3} = \dfrac{1}{3}$

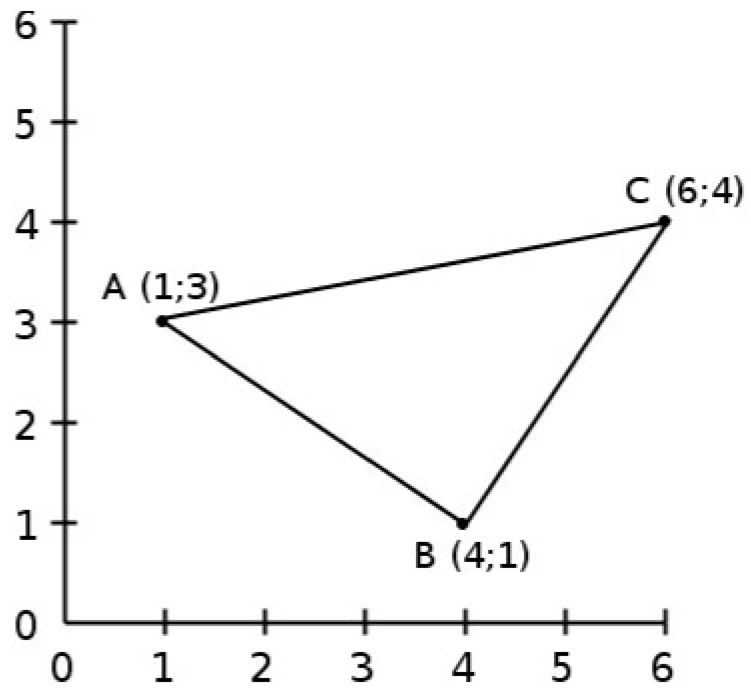
c) $Midpnt = \left(\dfrac{3}{2}\right); \left(-\dfrac{1}{2}\right)$

d) No, PQRS is not a parallelogram

8) $$\frac{-2+2}{2}=a$$
$$a=0$$

$$\frac{3+6}{2}=b$$
$$b=\frac{9}{2}$$

9)



b) $d_{AB}=\sqrt{(4-1)^2+(1-3)^2}=\sqrt{13}$
$d_{BC}=\sqrt{(6-4)^2+(4-1)^2}=\sqrt{13}$
$d_{AC}=\sqrt{(6-1)^2+(4-3)^2}=\sqrt{26}$
Two sides are equal and so the triangle is isoceles.
c) $M_{AC}=\left(\frac{1+6}{2};\frac{3+4}{2}\right)=\left(\frac{7}{2};\frac{7}{2}\right)$
d) $m_{AB}=\frac{1-3}{4-1}=\frac{-2}{3}$
e) $m_{BD}=\frac{-1-1}{7-4}=\frac{-2}{3}$
$m_{AD}=\frac{-1-3}{7-1}=\frac{-2}{3}$
So A, B and D are collinear.

10)

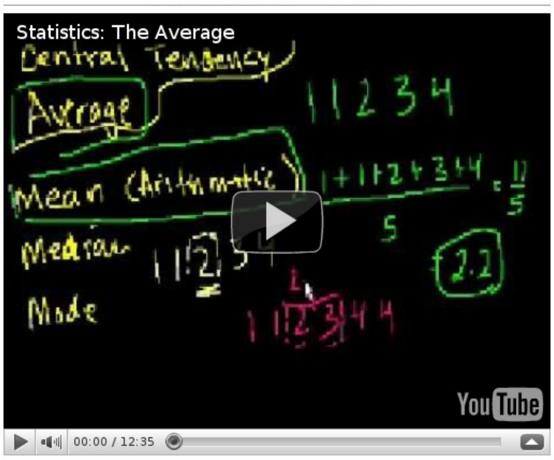
a) $AB{:}y=\frac{1}{3}x+3$
b) $AB=\sqrt{40}$
c) A' = (7;-3) and B' = (-1;1)
d) $A'B'{:}y=\frac{1}{3}x-\frac{2}{3}$
e) $A'B'=\sqrt{40}$
f) Yes, one pair of opposite sides is equal and parallel

Chapter 17 - Statistics

http://cnx.org/content/m38371/latest/?collection=col11306/latest

Videos

Khan academy video on statistics



This Khan Academy video on The Average covers the concepts of central tendency, namely mean, median and mode. This video starts off with a brief overview of statistics and the types of statistics, and then covers some examples of central tendency of a set of numbers.

This video can be downloaded from: http://www.fhsst.org/lC8

Solutions

Exercises – Grouping Data

1.

Group	Tally	Frequency
$130 \leqslant h < 140$	卌 ||	7
$140 \leqslant h < 150$	卌	5
$150 \leqslant h < 160$	卌 ||	7
$160 \leqslant h < 170$	卌 ||||	9
$170 \leqslant h < 180$	||	2

2.

Group	Frequency
11 – 20	7
21 - 30	13
31 - 40	15
41 - 50	8
51 - 60	7

Exercises – Summarising Data

1.

Each set is already ordered, so we do not need to order the sets.

a) The range is the difference between the highest and lowest values.

Data set 1: The highest value is 24 and the lowest value is 9. The range is $24-9=15$

Data set 2: The highest value is 16 and the lowest value is 7. The range is $16-7=9$

Data set 3: The highest value is 27 and the lowest value is 11. The range is $27-11=16$

b – f) The quartiles divide the data into four equal parts. We count how many values there are in each data set and then divide this by 4. This will mark out each data set into 4 equal parts and give us the quartiles. Note that the middle quartile is the median.

Data set 1: There are 7 values in the data set. $\frac{7}{4}=1{,}75$
So the lower quartile is 12, the median is 14 and the upper quartile is 22.

The interquartile range is $Q_3-Q_1=22-12=10$

The semi-interquartile range is $\frac{Q_3-Q_1}{2}=\frac{10}{2}=5$

Data set 2: There are 8 values in the data set. $\frac{8}{4}=2$
In this case we need to take the average of the two numbers that lie on either side of the split to get the quartile.

So the lower quartile is $\frac{7+8}{2}=7{,}5$, the median is $\frac{11+13}{2}=12$ and the upper quartile is $\frac{15+16}{2}=15{,}5$.
The interquartile range is $Q_3-Q_1=15{,}5-7{,}5=8$

The semi-interquartile range is $\frac{Q_3-Q_1}{2}=\frac{8}{2}=4$

Data set 3: There are 9 values in the data set. $\frac{9}{4}=2{,}25$
This is slightly more tricky to see how the numbers are equally divided. We will work out the median first. The median is the value at position $\frac{9+1}{2}=5$. This is 19.

$$11 \quad 15 \;\Big|\; 16 \quad 17 \quad \boxed{19} \quad 19 \quad 22 \;\Big|\; 24 \quad 27$$

median

The lower quartile is $\frac{15+16}{2}=15{,}5$ and the upper quartile is $\frac{22+24}{2}=23$.
The interquartile range is $Q_3-Q_1=23-15{,}5=7{,}5$

The semi-interquartile range is $\frac{Q_3-Q_1}{2}=\frac{7{,}5}{2}=3{,}75$

2.

a) The mean number in the first two jars is calculated by adding up the number in each jar and dividing it by 2. So the mean number in the first 3 jars is found by adding up the numbers in all 3 jars and dividing it by 3. This gives (using n_3 to represent the number in the 3rd jar)

$$\frac{1+3+n_3}{3}=3$$
$$1+3+n_3=9$$
$$n_3=5$$

b) Following the same process as in part a we get:

$$\frac{1+3+5+n_4}{4}=4$$
$$9+n_4=16$$
$$n_4=7$$

3.

We have 5 ages: x_1, x_2, x_3, x_4, x_5

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 5$$

The mean is $x_1 + x_2 + x_3 + x_4 + x_5 = 25$

The median value is at position $\frac{5+1}{2} = 3$. So $x_3 = 3$

The mode is the age that occurs most often. So we would expect at least 2 ages to be 2. The other ages would all have to be different. The four unknown ages can't all be 2, as that would not give us a mean of 5. Also since all calculations of mean, mode and median are done on ordered sets of data, we can't have 3 ages being 2, because then the median age would not be 3.

$$2 + 2 + 3 + x_4 + x_5 = 25$$

We have $18 = x_4 + x_5$. x_4, x_5 can be any numbers that add up to 18 and are not the same, so 12 and 6 or 8 and 10 or 3 and 15, etc. Learners could give any of the following:

2, 2, 3, 3, 15 or 2, 2, 3, 4, 14 or 2, 2, 3, 5, 13 or 2, 2, 3, 6, 12 or 2, 2, 3, 7, 11 or 2, 2, 3, 8, 10

Note that the set of ages must be ordered, the median value must be 3 and there must be 2 ages of 2.

4. We have $x_1, x_2, x_3 x_4$. We know that one of these numbers is 4. We also know that:

$$\frac{x_1 + x_2 + x_3 + x_4}{4} = 10$$

$$x_1 + x_2 + x_3 + x_4 = 40$$

Since we are given that the friend who left had 4 marbles, the total remaining marbles will be:

$$x_1 + x_2 + x_3 = 40 - 4 = 36$$

5. We first order the data:

3; 15; 16; 19; 23; 27; 27; 39; 43; 45; 54; 65

The minimum is 3 and the maximum is 65.

There are 12 data points, so the first quartile lies between 3 and 4:

$$\frac{16 + 19}{2} = 17{,}5$$

The second quartile lies between 6 and 7:

$$\frac{27 + 27}{2} = 27$$

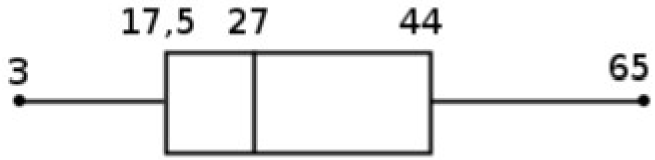
The third quartile lies between 9 and 10:

$$\frac{43 + 45}{2} = 44$$

The five-number summary is:

3; 17,5; 27; 44; 65.

The box and whisker plot is:



6. We first order the data:

1; 2; 12; 12; 19; 22; 35; 43; 45; 48; 49; 60

The minimum is 1 and the maximum is 60.

There are 12 data points, so the first quartile lies between 3 and 4:

$\frac{12+12}{2}=12$

The second quartile lies between 6 and 7:

$\frac{22+35}{2}=28{,}5$

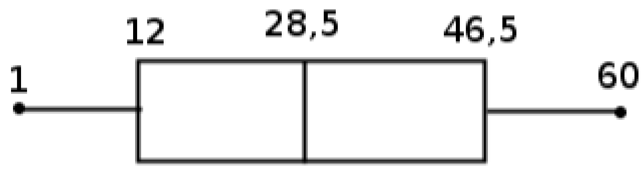
The third quartile lies between 9 and 10:

$\frac{45+48}{2}=46{,}5$

The five-number summary is:

1; 12; 28,5; 46,5; 60.

The box and whisker plot is:



7. a) We first order the data:

3; 5; 5; 6; 7; 8; 12; 14; 16

The highest value is 16 and the lowest value is 3.

There are 9 data points. So the first quartile occurs between the 2nd and 3rd data points:

$\frac{5+5}{2}=5$

The second quartile occurs at: 7

The third quartile occurs between the 7th and 8th data points:

$\frac{12+14}{2}=13$

The five-number summary is:

3; 5; 7; 13; 16

b) The second quartile does not lie between two data values, but instead occurs at just one data value.

More Mean, Modal and Median Group Exercises

1.

We will work out the modal group first as that is the easiest to work out. It is simply the group that occurs the most frequently. From the table we see that the modal group is 66 – 75.

The mean is slightly more difficult as we are dealing with grouped data.

Since we do not know the actual times we approximate the times by using the mid point of each range.

Time (s)	Midpoint	Frequency	Midpoint x Frequency
36 - 45	40,5	5	202,5
46 - 55	50,5	11	555,5
56 - 65	60,5	15	907,5
66 - 75	70,5	26	1833
76 - 85	80,5	19	1529,5
86 - 95	90,5	13	1176,5
96 - 105	100,5	6	603
		Total = 95	Total = 6807,5

The mean is $\frac{6807,5}{95}=71,66$

To work out the median group we take the total of the frequencies, add 1 and divide by 2. This gives $\frac{95+1}{2}=48$. Now we work out which group this falls into. The median group is 66 – 75 as this is where the 48th data value falls.

2.

We start with the modal group as this is the easiest to work out. The modal group is 61 – 65 since this has the highest frequency.

Mass (kg)	Midpoint	Frequency	Midpoint x Frequency
41 - 45	43	3	129
46 - 50	48	5	240
51 - 55	53	8	424
56 - 60	58	12	696
61 - 65	63	14	882
66 - 70	68	9	612
71 - 75	73	7	511
76 - 80	78	2	156
		Total = 60	Total = 3650

The mean is $\frac{3650}{60}=60,83=61$

To work out the median group we take the total of the frequencies, add 1 and divide by 2. This gives $\frac{60+1}{2}=30,5$. Now we work out which group the 30th and 31st data values fall into. The median group is 61 – 65 as this is where the 30th and 31st data values fall.

Exercises – Misuse of Statistics

1.

The graph should not convince learners that the companies earnings have increased. The vertical axis is not labelled which means that we do not know by how much exactly the earnings increased. We also cannot tell if the two bars in the chart actually represent earnings since there is no key to tell us what the graph is about. Also since the vertical axis is not labelled we cannot tell if the company has zoomed in on the chart to show a big increase when actually there is very little increase.

2.

No. Although the number of white cars that are speeding have a higher frequency than blue and red cars, we cannot conclude that drivers of white cars speed more than drivers of red or blue cars. All the data tells us is that there were more white cars found speeding than red or blue cars. To determine if drivers of white cars are more likely to break the speed limit, we would also have to look at the number of cars of each colour. We do not know if there are simply more white cars than red or blue cars.

3.

The record label used a bigger block (wider) to show their sales. This gives the visual impression that their sales were bigger.

They also used pictures of stick men. The one in their block looks happier than the one in the competitors block.

By not placing a line on the y-axis we cannot determine exactly where the 40 and 50 million copies occur, we rely on our eye to guide us and assume that there are two values, and two blocks so each value corresponds to a block.

4.

Although there is a drop in the number of passengers from December on, this would be expected as the tourist season is in summer, so the winter months would have less passengers.

Learners should see that there is insufficient evidence here to show that the competitor is losing business.

Learners should notice that this graph is incomplete and so we have no way of knowing what happened from July to September.

5.

Although noisy office environments appear to be linked to poor productivity we cannot conclude that there is a definite link from the graph. The y-axis needs to be labelled to see over what range the productivity changed. More samples could also be taken. Also there are many other factors which may lead to poor productivity and so we cannot conclude that the only cause of poor productivity is a noisy office environment. All we can really say is that there appears to be a link between noise and productivity.

Exercises

1.

There are 70 data values. The ordered data values are:

776.71	789	793.63	799.01	801.82	806.65	812.62
780.23	789.08	795.21	799.05	802.05	807.32	815.63
780.38	789.24	795.86	799.35	802.2	807.41	815.74
784.68	789.45	796.2	799.35	802.37	807.89	817.57
785.37	790.69	796.33	799.84	802.39	808.8	817.76
786.59	790.83	796.67	801.01	802.5	809.05	818.26
787.65	791.13	796.76	801.21	803.16	809.3	819.54
787.78	791.23	797.72	801.24	805.28	809.68	819.59
787.87	792.19	798.72	801.45	805.99	809.8	820.39
788.99	792.43	798.93	801.48	806.64	812.61	825.96

To work out the mean we need the sum of these values. The mean is:

$\frac{56027}{70}=800{,}39$

The median is at position 35,5. So we take the average of the value at position 35 and 36. This is

$\frac{799{,}84+801{,}01}{2}=800{,}43$

To work out the mode we need to find which value occurs the most often. We can do this either by just looking through the ordered set and seeing if any number occurs more than once, or we can draw up a frequency table. From looking through the ordered set we see that 799,35 occurs twice, while all other values occur only once. The mode is 799,35.

To summarize:

The mean is 800,39. The mode is 799,35 and the median is 800,43.

2. We first order the data set.

   41, 42, 42, 44, , 47, 47, 60

   The median is at position 4. This is 44.

3. We first order the data set:

   4, 4, 5, 5, 6, 6, 6, 6, 7, 7

   From this we see that the most common data value is 6, and so the mode is 6.

4.

a) The mean and the mode. The mean will give us the time that it takes on average, while the mode will give us the time that is most often obtained.

b) The mean for bike 1 is 1,02 s

The mean for bike 2 is 0,98 s (or 1,0 s)

For bike 1 there is no value that occurs more than once.

For bike 2 1,0 and 0,9 both occur four times, so the mode would be 0,95.

c) It would be difficult to choose. Although bike 1 appears to do better than bike 2 from the mean, the data for bike 2 is less accurate than that for bike 1 (it only has one decimal place.) If we were to calculate the mean for bike 1 using only 1 decimal place we would get 0,9 s. This would make bike 2 better. Also bike 2 produces more consistent numbers. So bike 2 would likely be a good choice, but more information or more accurate information should be obtained.

5.

a) We need to determine the intervals so we order the data and look at the highest and lowest values. The lowest value is 131 and the highest value is 173. The range is $173-131=42$. The interval width is $\frac{42}{6}=7$. So the intervals are 131 – 137, 138 – 144, 145 – 151, 152 – 158, 159 – 165, 166 – 172. (Each interval must start 7 numbers after the previous interval, i.e. 131 + 7 = 138 and so the interval ends one number less than that).

Group	Frequency
131 - 137	4
138 - 144	7
145 - 151	10
152 - 158	6
159 - 165	6
166 - 173	7

b) The actual mean is $\frac{6062}{40}=151,6$.

Group	Midpoint	Frequency	Midpoint x Frequency
131 - 137	134	4	536
138 - 144	141	7	987
145 - 151	148	10	1480
152 - 158	155	6	930
159 - 165	162	6	972
166 - 173	169	7	1183
		Total = 40	Total = 6088

The approximate mean is $\frac{6088}{40}=152,2$

Notice that the approximate mean is calculated from the grouped data while the actual mean is calculated from all the data. The two values are very similar.

c) The modal group is 145 – 151 cm. This is the group that occurs the most often. There are three values that occur three times. These are 132, 150 and 152. Notice how two of these values fall into the modal group.

d) The average is 152. The group that the average occurs in is 152 – 158. There are 16 learners who are taller than the average. However there are only 13 that are in the two groups above the average group.

6.

a)

Distance (km)	Midpoint	Frequency	Midpoint x Frequency
1 – 5	3	4	12
6 – 10	8	5	40
11 – 15	13	9	117
16 – 20	18	10	180
21 – 25	23	7	161
26 – 30	28	8	224
31 – 35	33	3	99
36 – 40	38	2	76
41 – 45	43	2	86
		Total = 50	Total = 995

The approximate mean is $\frac{995}{50}=19{,}9$

bi) There were 18 drivers who drove less than 16 km. The percentage is $\frac{18}{50}\times100=36\,\%$

ii) There were 7 drivers who drove more than 30 km. The percentage is $\frac{7}{50}\times100=14\,\%$

iii) $100-36-14=50\,\%$. We could have also said that there were 25 drivers and so the percentage is $\frac{25}{50}\times100=50\,\%$

7.

a) We must first order the data sets for both trained and untrained.

Trained: 118, 120, 121, 125, 126, 127, 128, 129, 130, 130, 131, 132, 134, 135, 137

Untrained: 126, 134, 135, 139, 140, 142, 142, 144, 145, 145, 148, 149, 152, 153, 156

There are 15 values in each data set. The median is at position $\frac{15+1}{2}=8$. For the trained workers this is 129 and for the untrained workers this is 144.

The positions of the quartiles are $\frac{15}{4}=3{,}75$.

Trained: Lower quartile is 125 and upper quartile is 132

Untrained: Lower quartile is 139 and upper quartile is 149

b) The interquartile range for trained is:

$Q_3-Q_1=132-125=7$

The interquartile range for untrained is:

$Q_3-Q_1=149-139=10$

c) The median of the untrained workers is higher than that of the trained workers. Also the untrained workers have a larger interquartile range than the trained workers. There is some evidence to suggest that the training programme may be working.

8.

a) The mean is $\frac{1640000}{9}=182222{,}22$

b) The mode is R 100 000.

c) To find the median we need to order the data. To make the numbers easier to work with we will divide each one by 100 000. The ordered set is 80, 90, 100, 100, 100, 120, 200, 250, 600. The median is at position 5 and is R 100 000.

d) Either the mode or the median. The mean is skewed (shifted) by the one salary of R 600 000. The mode gives us a better estimate of what the workers are actually earning. The median also gives us a fairly accurate representation of what the workers are earning.

9.

Class	Tally	Frequency	Mid-point	Frequency x Midpoint
30 - 39	11	2	34,5	69
40 - 49	1	1	44,5	44,5
50 - 59	111	3	54,5	163,5
60 - 69	~~1111~~ 1111	9	64,5	580,5
70 - 79	~~1111~~ 1	6	74,5	447
80 - 89	~~1111~~	5	84,5	422,5
90 - 99	11	2	94,5	189
		Total = 28		Total = 1916